Quantization Effects in Radiation Spectroscopy Based on Digital Pulse Processing

Valentin T. Jordanov, *Senior Member, IEEE.*, and Kalina V. Jordanova

***Abstract*–Radiation spectra represent inherently quantization data in the form of stacked channels of equal width. The spectrum is an experimental measurement of the discrete probability density function (PDF) of the detector pulse heights. The quantization granularity of the spectra depends on the total number of channels covering the full range of pulse heights. In analog pulse processing the total number of channels is equal to the total digital values produced by a spectroscopy analog-to-digital converter (ADC). In digital pulse processing each detector pulse is sampled and quantized by a fast ADC producing certain number of quantized numerical values. These digital values are linearly processed to obtain a digital quantity representing the peak of the digitally shaped pulse. Using digital pulse processing it is possible to acquire a spectrum with the total number of channels greater than the number of ADC values. Noise and sample averaging are important in the transformation of ADC quantized data into spectral quantized data. Analysis of this transformation is performed using an area sampling model of quantization. Spectrum differential nonlinearity (DNL) is shown to be related to the quantization at low noise levels and small number of averaged samples. Theoretical analysis and experimental measurements are used to obtain the condition to minimize the DNL due to quantization.**

I. Introduction

Recently, we have developed a concept of a radiation spectrometer that acquires and stores radiation spectra at the highest possible total number of channels per spectrum. That is, for example, all spectra are acquired and stored as *32k* (2^{15}) channel spectra despite the analog-to-digital converter resolution used by the spectrometer. At any time the high channel count (e.g. *32k*) spectrum can be transformed by software to a new spectrum with a lower number of channels without any loss of information while copy of the original spectrum is preserved. Such a transformation is straightforward and could eliminate the outdated hardware "conversion gain" setting. Furthermore, it would give the user the flexibility to view and choose channel resolution [1,2] with a single spectrum acquisition. Technically, this is easily achievable due to the abundance of storage memory and readily available software routines in modern microprocessor-based spectrometers.

In the classic analog spectrometers the maximum achievable channel count per spectrum is equal to the number of the quantization levels of the analog-to-digital converter (ADC), which is typically 2^N, where N is the number of ADC bits. Thus, if the ADC has 13 bits than the storage of *8k* (2^{13}) channels into the spectrum space of *32k* channels will require either a multiplication by a factor of four of the ADC digital values or artificially spreading out the counts of one channel over four channels. In the first case the resulting *32k* spectrum will have differential nonlinearity of 100% since every 3 of 4 consecutive channels will always have zero counts. In the second case there will be distortion of data for the *32k* spectrum. However, in both cases spectra can be binary scaled down to *8k* spectrum identical to the original *8k* spectrum.

In the radiation spectroscopy based on digital pulse processing (DPP), however, the number of total spectrum channels can be grater than the maximum number of ADC quantization levels without introducing the effects of spreading the data over more channels, namely the 100% differential nonlinearity or distorting the spectra as in the case of the classic analog spectroscopy. In this paper we attempt to present the quantization effects and conditions which allow such an improvement of the channel resolution. This discussion relates to the specifics of radiation spectroscopy and may not be applicable to other fields.

II. Quantization

A. Quantizer

The quantization is a nonlinear operation which converts continuous physical quantities into numerical values. The numerical values are a non-physical (energy-less) representation of the physical quantity. In this work we adopt some of the definitions and annotations defined in reference [3]. The mathematical operator that performs quantization is called a quantizer. In the field of signal processing the quantizer transforms an input signal x into numerical values x' by assigning to each numerical value a certain amplitude range of the continuous input signal x. The range of the input signal corresponding to a single numerical value is the quantum size q, also referred to as the basic unit of quantization or quantum step size. If q is the same (constant) for all numerical values over the entire range of the quantized input signal then the quantization is uniform. Thus, the differential nonlinearity of a uniform quantizer is equal to zero. In the analysis of this work we will consider a uniform quantizer.

By definition there is always a difference between the quantizer input signal x and the quantizer output signal x'. This difference is often referred to as quantization error. Quantization error is systematic, uniquely defined and fully predictable for a given quantizer. Therefore, the quantization error can not be treated the same way as random noise [3, 4].

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Nevertheless, the subject of quantization error and quantization noise has been extensively studied for more than half a century [5] due to of the unique relationship between sampling, quantization and digital signal processing.

B. Sampling and Analog-to-Digital Conversion

Sampling in signal processing is the operation of capturing instant values (samples) of the input signal. Note that the sampled values are physical quantities. The sampled values are quantized resulting in numerical values corresponding to the values of the sampled signal. This process of sampling and the associated quantization is well known as analog-to-digital conversion. Analog-to-digital conversion is an important technique making it possible to use mathematical operations on a digitized signal to perform non-physical signal processing: digital signal processing. In this work we consider quantization as a result of analog-to-digital conversion generating integer numbers with quantum size of one.

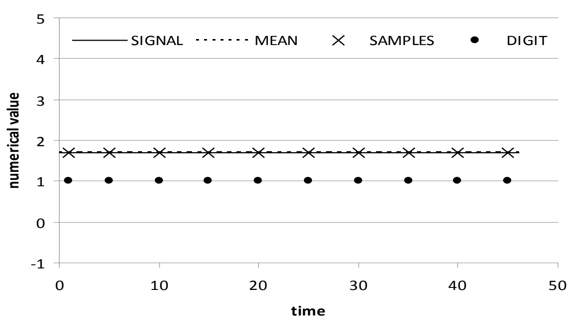
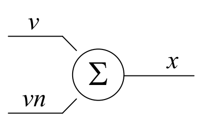
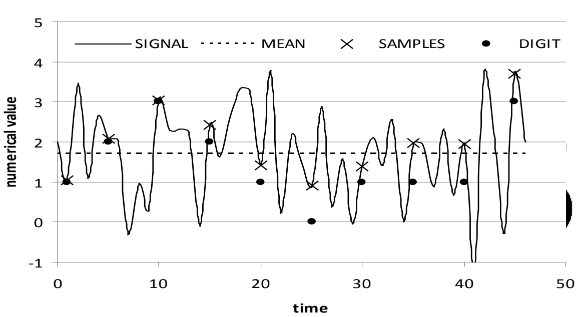


Fig. 1 Measurement of constant voltage using ADC samples.

Fig. 1 shows a measurement example of a constant, noiseless voltage of 1.7V. The voltage signal is digitized using an ADC with a quantum size of 1V. The input signal of 1.7V is sampled and quantized 10 times over a given period of time. The quantization output is “1” for all samples. As the quantum size is 1V the systematic quantization error is 0.7V for all samples. The average of these ten samples also has a quantization error of 0.7V.



a)



b)

Fig. 2 Measurement of constant voltage in presence of noise.

A more realistic signal is shown in Fig. 2, in which the signal is accompanied by noise. Sources of noise in the signal are: the generators of the signals, the signal conditioning electronics, the ADC and other internal and external sources coupled to the measurement system.

The signal x at the input of the ADC can be represented as a superposition of two components – measured noiseless signal (signal of interest) v and noise vn as depicted in Fig. 2a. Fig. 2b shows an example of analog-to-digital conversion of the same 1.7V signal in the presence of noise. In this example white noise is added to the measured constant signal of 1.7V. The quantization error of each sample is between 0 and 1V as expected from the definition of the quantization. The average of the ten samples is 1.5V which indicates a 0.2V measurement error of the average. Thus, the topic we will be discussing is the cause of the reduced quantization error of the measurement with noise compared to the case without noise – an observation made and question raised by Lakatos in his early work on digital signal processing in radiation measurement [6].

The sampling and quantization operations are independent from the input signal. Therefore, the quantizer will always convert 1.7V to a digital “1” regardless of the input signal composition. It is important to note that the properties of the ADC are independent from the presence of noise or its level relative to the measured signal. The reduced quantization error of the average of the ten samples is due to the mathematical operation of averaging and should not be regarded as an improved ADC resolution. In this paper we will are going to explore the effect of averaging, weighted or conventional, on the quantization error of the pulse-height measurement in the presence of noise. This analysis uses a statistical approach to address the random noise effects on the quantization and the measurement as a whole.

III. AREA SAMPLING

Consider a signal x that is a superposition of random white noise vn with zero mean added to a noiseless measured constant value of interest v as shown in Fig. 2b. The measurement goal is to estimate v using the average of multiple quantized samples of the “noisy” signal x. Obviously, the quantized output will be spread over multiple numerical values x' because of the randomness of the signal x which can be viewed mathematically as a random variable. The random variable x is characterized by a probability density function (PDF) $f_x(x)$ which has the same shape as the PDF of the noise and mean equal to the signal of interest v. Multiple measurements of x will yield a quantized output x' which is also a random variable with PDF $f_{x'}(x')$. The PDF $f_{x'}(x')$ determines the uncertainty of the measurement of v and depends on the quantization parameters as well as the PDF $f_x(x)$ of the input signal x. The relationship between $f_x(x)$ and $f_{x'}(x')$ can be obtained from the definition of quantization.

Let the quantum size q be such that the range x_n - $0.5q$ to x_n - + $0.5q$ of the input signal x is quantized to a digit n of the quantization output x' as shown in Fig. 3. In other words, x' is the quantized measure of x in units q.

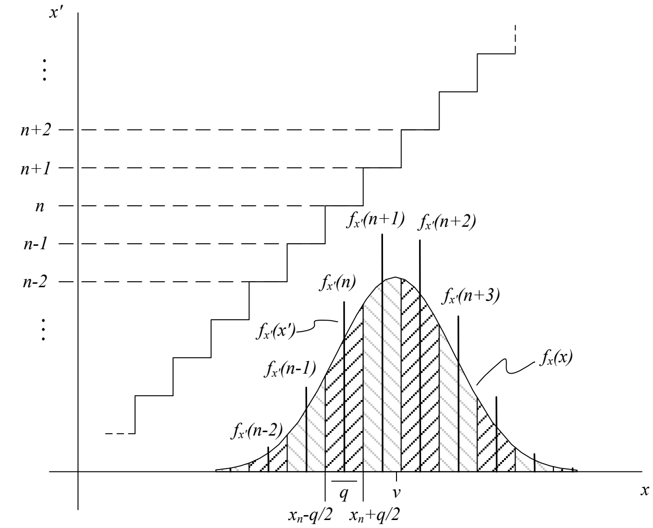


Fig. 3 Probability density function transformation in the process of quantization.

The probability that the input signal x will be quantized into quantized output $x'=n$ will be the probability that x takes any of the values between $x_n - 0.5q$ and $x_n + 0.5q$.

The probability of a quantization outcome $x'=n$ in repeated quantization trials of the measured signal x is the area (integral) of the PDF $f_x(x)$ over the interval $x_n - 0.5q$ to $x_n + 0.5q$. As the integral is a single value the continuous PDF $f_x(x)$ is transformed into a discrete PDF $f_{x'}(x')$.The probability that v will be quantized into the digit n is:

$$f_{x'}(n) = \int_{x_n - 0.5q}^{x_n + 0.5q} f_x(x)\,dx \tag{1}$$

Similarly, the probabilities that the signal of interest v will be converted to other quantized values can be obtained by integrating the PDF $f_x(x)$ over the corresponding quantization interval with width q. Thus, the quantization, in statistical context, can be viewed as an operation that transforms the continuous PDF of the input x to discrete PDF of the quantized output x'. This transformation and the underlying theory has been developed by Widrow [3, p.27, and references therein] and is referred to as “area sampling”.

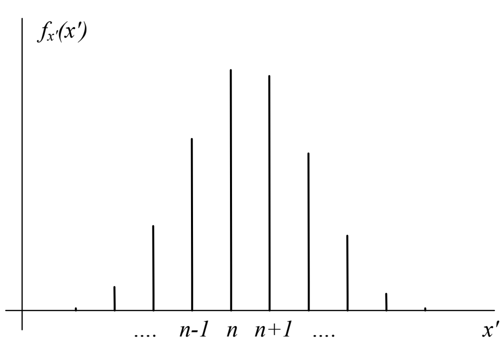


Fig. 4 Discrete probability density function of the quantized output.

The function $f_{x'}(x')$ is a discrete probability density function but can be also considered a continuous PDF $f_{x'}(x')$ consisting of a sequence of Dirac impulses spaced at unit intervals and weighted by the area as defined by Eq.1.

The discrete probability density function depends on the signal of interest v and for every v there is a corresponding PDF of the quantization output x'. Thus, the probability that a measured value v will produce a quantization output x' is defined by the PDF $f_{x'}(x',v)$. Note that v is not a random variable. Fig. 5 depicts the $f_{x'}(x',v)$ as function of v in a single quantization interval q. The noise vn in this example is Gaussian with standard deviation $\sigma = 1.5q$. The PDF pattern of $f_{x'}(x',v)$ will cyclically repeat with period of q, that is $p(x'=i, v+iq) = p(x'=j, v+jq)$ for every $i, j \in x'$. Some important properties of the discrete PDF $f_{x'}(x',v)$ are worth mentioning. At both ends of the quantization interval $-0.5q$ and $0.5q$ the neighboring digits (e.g. 0 and 1; 0 and -1) are equally probable as expected from the definition of quantization. The highest probability for v to be converted to any numerical value is when $v=iq$ (middle of each quantization interval): $p(x'=i, iq) = m$ for every $i \in x'$. The dependence of $f_{x'}(x',v)$ on may v may cause differential nonlinearity and/or distortion in DPP acquired spectra.

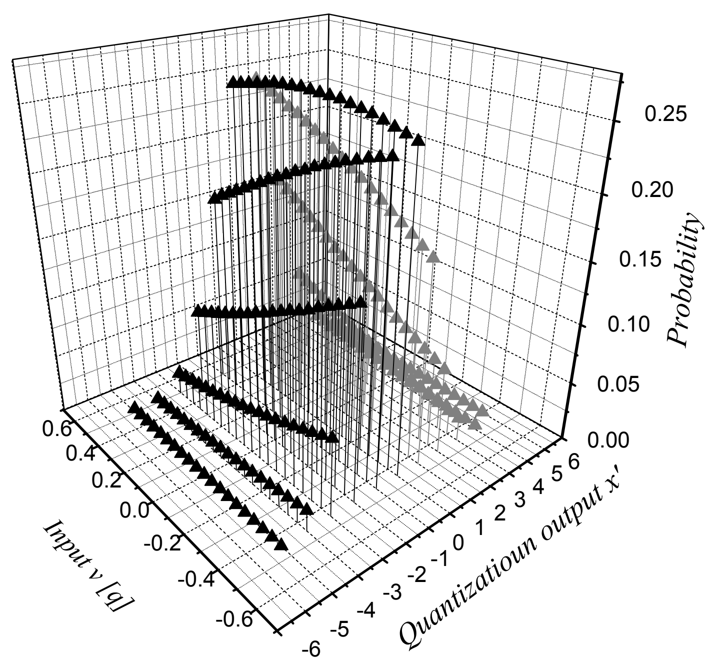


Fig 5 Discrete PDF of the quantization output

For every v the discrete PDF $f_{x'}(x',v)$ has mean

$$\mu_{x'}[v] = \sum_i i \cdot f_{x'}[i,v] \tag{2}$$

and variance

$$\sigma_{x'}{}^2[v] = \sum_i (i - \mu[v])^2 \cdot f_{x'}[i,v] \tag{3}$$

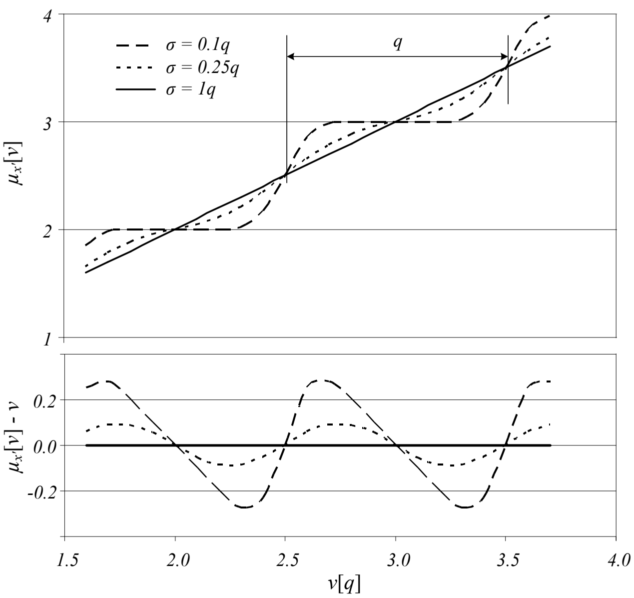


Fig 6 Mean of the discrete PDF (top) an the error (bottom) relative to the continuous PDF.

Fig 6 shows a plot of the mean (Eq. 2) of the discrete PDF $f_{x'}(x',v)$ for different noise contributions of vn. The mean is the measure of v when a very large number of measurement trials is carried out. Thus, the measurement error can be found by the difference between $\mu_{x'}[v]$ and v as depicted in Fig. 6. Our numerical simulation indicates that a noise vn with $\sigma \geq q$ makes this error diminish.

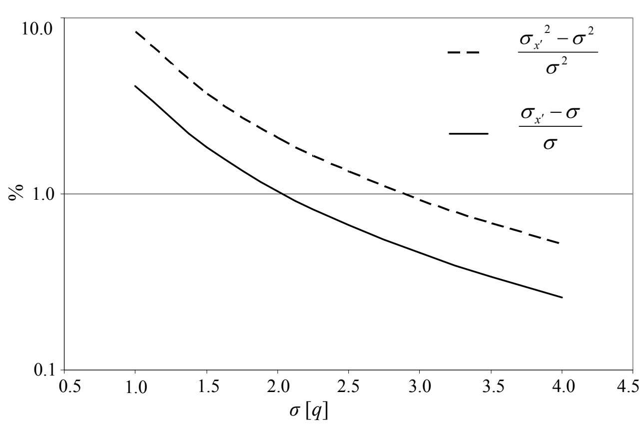


Fig 7 Relative change of the variance and the standard deviation of discrete PDF in the presence of Gaussian noise.

Combined, the area sampling definition and Eq. 3 indicate that the variance of the quantized output x', $\sigma_{x'}^2[v]$, increases proportionally to the increase of the variance, σ^2 of the noise vn. If $\sigma \geq q$ than $\sigma_{x'}^2$ depends very little on v and is only a function of σ. Fig. 7 shows the relative increase of the variance $\sigma_{x'}^2$ and the standard deviation σ_x due to quantization. At high noise levels relative to the quantum size the noise contribution dominates and reduces the effect of the quantization on the overall measurement uncertainty. It will be shown experimentally that the σ/q ratio is critical for reducing the quantization effects on the spectrum DNL and the spectral peak resolution. This reduction is primarily due to the improved estimation accuracy of the mean of the measured pulse heights obtained as a weighted average over multiple quantized values.

IV. Pulse-Height Analysis

Radiation spectroscopy is a complex measurement technique with the ultimate goal of determining sources of radiation and their strength. This measurement comprises different components, a radiation detector being the most important. Spectroscopy radiation detectors produce signals that are proportional to the energy deposited by ionizing particles interacting with the detector. The signal corresponding to a single interaction in the detector usually has a short duration. Semi-conductor detectors and gas-filled spectroscopy detectors respond to particle interaction by producing current pulses with typical durations from a few nanoseconds to a few microseconds. In general, the total charge carried by these current pulses is proportional to the energy deposited in the detector. Scintillation detectors respond by emitting light as exponentially decaying pulses with decay time constants typically in the nanosecond-microsecond range. The total light emitted is a measure of the energy deposited into the scintillation detector. The generation of the detector signals is subject to statistical variations due to the random nature of charge transport or scintillation light emission.

Measurement of the detector pulses is further complicated by the presence of electronic random noise. This noise is generated independently from the generation of the detector signal. Thus, noise is always present in a spectroscopy system. To reduce the effect of the noise on the spectroscopy measurement, noise filtering techniques are commonly used. Noise filters are often called pulse shapers. Pulse shapers transform the detector current pulses into voltage or digital pulses. The pulse shapers are in most instances linear systems that produce pulses whose peak amplitudes are proportional to the energy deposited into the radiation detector.

Radiation spectroscopy requires a plurality of detector pulses to estimate the energy deposition into the detector. The time sequence of the detector interactions is a random process. Therefore, radiation spectrometers process a random sequence of pulses whose peak amplitudes carry information about the energy deposited into the detector.

The spectroscopy information is derived from a measurement of the peak amplitudes (pulse-height) of the shaped detector pulses, a measurement often referred to as pulse-height analysis (PHA). It is important to note that the spectroscopy information is carried by the pulse heights and requires no knowledge and/or measurement of the frequency components of the signals. This is a major difference from other signal processing fields such as communication, speech recognition, video transmission etc. In these applications the frequency is a major component of the information carried by the signals. Sampling and quantization theories have been extensively developed for these fields and may be confusing or of little value when applied to the pulse-height analysis. For example, frequency aliasing does not in general affect the pulse-height measurement. Thus, implementation of anti-aliasing filters makes little sense and in some instances, may degrade the accuracy of the pulse height measurement by reducing the magnitude and the bandwidth of the noise.

Classical spectroscopy systems use analog shaping techniques and peak detector-stretcher to sample the pulse heights. The peak values are then digitized by an ADC. In this case a single sample-quantization operation is performed for each shaped detector pulse. The quantization result is used to address a spectrum channel. Normally a channel is assigned to each discrete value of the ADC. Channels are counters that count the number of each ADC value occurrence for the duration of the spectrum acquisition. Thus, the channels of the radiation spectra are quantized data. Each channel corresponds to a certain range of pulse heights called the channel width. The channel width is the quantum size of the spectrum. In the classical spectroscopy system the channel width is equal to the quantum size of the ADC. Thus, radiation spectra are inherently quantized data. In fact, radiation spectra represent experimental, area sampled discrete PDF as defined by Eq. 1.

Radiation spectroscopy based on DPP utilizes fast sampling ADCs. Pre-shaped pulses are sampled and quantized (digitized) and the final phase of the shaping is performed in the digital domain. Both time-invariant and time-variant linear digital filters can be used. As in the case of classic spectroscopy single point amplitude (digital peak value) measurement is performed to determine the corresponding spectroscopy channel. The digital peak is a weighted sum of the digitized signal values normalized so that the measured amplitude range of the pre-filtered signal generates digital peak values corresponding to a certain range of spectroscopy channels. As stated earlier the quantum size of the spectrum is not necessarily the same as the quantum size of the ADC. In other words the digital pulse processing in radiation spectroscopy may transform quantized data with one quantum size into quantized data with a different quantum size.

V. Pulse-Height Quantization

We will limit our analysis to a practical case of linear digital pulse shapers with finite impulse response (FIR) [7]. Let us define a FIR, h', of a time invariant digital pulse shaper with k coefficients $h'[i]$ from 0 to $k-1$ ($0 \leq i \leq k-1$), where $h'[0] \neq 0$, $h'[k-1] \neq 0$ and $h'[i]=0$ for $i<0$ *and* $i>k-1$. Let $v[i]$ be the noiseless instant value of the detector signal applied together with the noise vn and sampled as $x[i]$. As $x[i] = v[i] + vn$ by definition we will use $x'[i]$ to indicate the quantization of $v[i]$ in the presence of noise vn. The digital shaper response y' to digital signal x' can be expressed as a convolution sum

$$y'[j] = \sum_{i=-\infty}^{j} h'[j-i] \cdot x'[i] \tag{4}$$

If the peak value of a single, digitally shaped pulse occurs $p+1$ samples after an interaction in the radiation detector at time 0 ($x'[i] = 0$ for $i<0$), the peak value can be obtained from Eq. 4

$$y'[p] = \sum_{i=p+1-k}^{p} h'[p-i] \cdot x'[i] \tag{5}$$

This simple equation is the mathematical expression of the pulse-height measurement in radiation spectroscopy based on FIR time invariant digital pulse shapers. The equation is similar for time variant digital shapers, except that the summation begins from the sample at which the shaper switches its impulse response to process the incoming detector pulse.

It is clear that $y'[p]$ is a random variable because it is a result of a weighted sum of random variables $x'[i]$. In most cases the peaking interval p+1 is the same for all pulses, so we will use y' in place of $y'[p]$. The multiplication by constant a of the discrete random variable $x'[i]$ will create a new random variable $w'_a[i] = ax'[i]$ with PDF $f_{ax}(a\,x')$ as shown in Fig. 8.

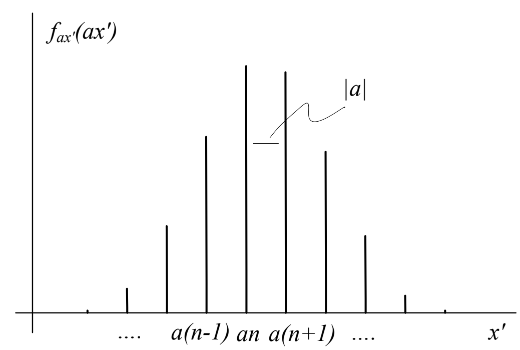


Fig. 8 Discrete PDF transformation by constant multiplication.

Let us make the following assignment in Eq. 5 $w'[i] = h'[p-i] \cdot x'[i]$. To obtain the PDF $f_{y'}(y')$ we will make the assumption that all samples $x'[i]$ are uncorrelated. The PDF of the peak measurement will be the convolution of all PDF $f_{w'}(w'[i])$ of the random variables $w'[i]$:

$$f_{y'}(y') = (\ldots(f_{w'}(w'[-(k-1)]) * f_{w'}(w'[-(k-1)])) * \ldots) * f_{w'}(w'[p]) \tag{6}$$

where "*" denotes convolution.

Analytical expression for $f_{y'}(y')$ may not be possible or will require a tedious mathematical exercise. Numerical computation is more suitable to calculate the PDF but a special attention should be paid to the numerical precision. Nevertheless, some important properties of the pulse height PDF $f_{y'}(y')$ can be identified from Eq. 4:

1) The pulse height PDF $f_{y'}(y')$ is a discrete PDF.
2) If all nonzero $|h'|$ values are equal or greater than one then the variance of the pulse height PDF is greater than the variance of any PDF of the digital samples $x'[i]$.
3) If all h' coefficients are integers then the minimum separation of the pulse height PDF nonzero values is equal to the greatest common divisor of all h' coefficients.

4) If all nonzero $|h'|$ are equal then the minimum separation is equal to $|h'|$. This is the case of simple averaging or box car averaging.

Properties 3) and 4) along with the gain normalization factor of the digital filter determine the quantum size of the pulse height measurement when properly scaled to match the energy range. Note that, this quantum size determines the maximum number of achievable channels with non-zero probability to record counts. There is a reduced ability to improve the spectrum quantum size when the FIR impulse response has a small number of coefficients and/or the variance of the noise is small compared to the quantum size of the ADC. In such circumstances the spectrum exhibits large differential nonlinearity. In fact, if the FIR impulse response has only one coefficient the quantum size of the spectrum will be less than or at most equal to the quantum size of the ADC – this is the case of classic analog based PHA.

To illustrate the reduction of the quantum size of the pulse-height measurement we will consider a time-variant FIR filter which averages N quantized samples of a step input signal. This is a case similar to the example illustrated in Fig. 2b. Let all $h' = 1$ and $q=1$. First, let the channel width be equal to the quantum size of the ADC. In this case the averaging filter has digital gain N. Using Eq. 5 the peak value (average value) can be expressed as:

$$y' = \frac{\sum_{i=0}^{N-1} x'[i]}{N} = \frac{S'}{N} \tag{7}$$

S' is a random variable with discrete PDF $f_{S'}(S')$ which will have a unit quantum granularity. From the definition of convolution the mean $\mu_{y'} = S'/N = \mu_{x'}$ which is one-to-one mapping of the ADC quantization output to the spectrum channel numbers. The quantization granularity of the discrete PDF $f_{y'}(y')$, however, is $1/N$. Let constrain y' to take only integer values so that the peak values can be used to address the corresponding spectrum channels. The division S' by N can be viewed as a transformation from a discrete domain with quantum size $1/N$ to the discrete domain of the spectrum channels with quantum size 1. This transformation is very similar to quantization of continuous signal except that a range of discrete values within the quantum size is assigned to a single digit. In this example N values within $S'/N-0.5$ and $S'/N-0.5$ will result in a single digit of y'. In similar fashion as area sampling a single value of the discrete pulse-height PDF $f_{y'}(y')$ will be the sum of N consecutive discrete values of the discrete PDF $f_{S'}(S')/N$.

Next we increase the digital resolution by factor of $K > 1$. Eq. 7 transforms to

$$y' = \frac{K\sum_{i=0}^{N-1} x'[i]}{N} = \frac{KS'}{N} \tag{8}$$

The digital resolution gain comes with gain of the pulse-height mean $\mu_{y'} = K\mu_{x'}$, while the quantum size of y' is preserved. Because N is always integer and $f_{S'}(S')$ is discrete PDF with the same quantum granularity as $f_{y'}(y')$, quantum size transformation requires that N/K be integer to avoid introduction of differential nonlinearity.

When $N=1$ there is no possibility to increase spectral digital resolution without introducing 100% differential nonlinearity. At $N=2$ it will be possible to double the number of channels relative to the number of the ADC discrete values.

In view of the early discussions about the mean and the variance of discrete PDF it is important to maintain the noise level high enough to achieve broad pulse-height PDF. The broadening of the pulse-height PDF also depends on the number of the averaged values. At lower noise levels and small number of averaged values the pulse-height PDF may not be adequate to provide a uniform probability for all pulse-heights over the full ADC input range.

From Eq. 5 and Eq. 6 the mean and the variance of the peak value can be obtained. If the variance of the noise is large enough then we can express the mean and the variance of the peak value as

$$\sigma_p^{\,2} = \sigma_{x'}^{\,2} \sum_{i=p+1-k}^{p} h'[p-i] \tag{9}$$

As expected the variance is independent of the measured signal. It only depends on the variance of the quantized output which is furthermore a function of the input noise vn. This allows us to conduct experiments with uniformly distributed pulse-heights to study the spectrum differential nonlinearity independent of the noise level.

VI. Experimental Data

The goal of the experiments was to study the effect of the discrete PDF $f_{y'}(y')$ and the noise vn on the differential nonlinearity of the spectrum with quantum size smaller than the quantum size of the ADC. Fig. 9 shows a simplified block diagram of the experiment.

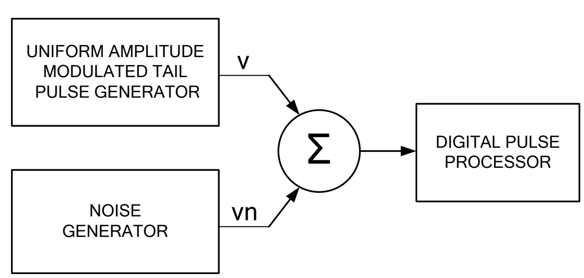


Fig. 9 Discrete PDF transformation by constant multiplication.

A tail pulse generator produces exponential pulses with decay time constant of 5μs and uniformly distributed amplitudes. Noise from the noise generator is added to the tail pulses and the resulting signal is applied to a digital pulse processor implementing a digital triangular shaper as described in [7]. The tail pulses are sampled at 80MHz by a 14-bit ADC with the lower 6 bits masked resulting in an 8-bit quantization. This reduces the relative noise contribution of the amplifiers, the ADC itself, etc. This bit reduction also significantly reduces the contribution of the ADC differential nonlinearity. Fig. 10 shows test pulses at different noise levels (standard deviation σ) relative to the quantum size q of the 8-bit ADC.

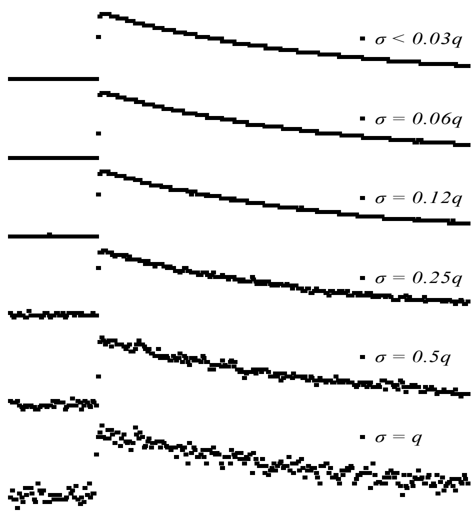


Fig. 10 Exponential pulses with different noise contribution.

Series of spectra were acquired using the full bandwidth of the noise generator at different settings of the digital pulse shaper and at different noise levels. Spectra were acquired at quantization gain of 32 – *8k* channels from an 8-bit ADC. However, the differential nonlinearity was estimated from scaled down *1k* spectra, which is a more realistic case. The results are summarized in Fig. 11.

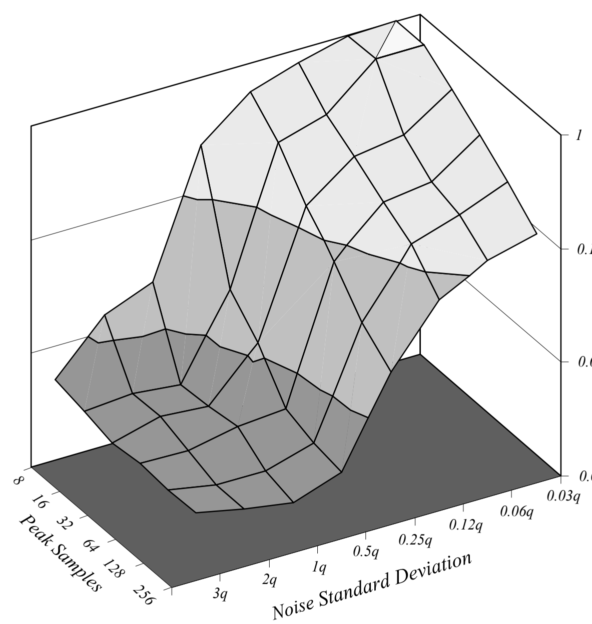


Fig. 11. Differential nonlinearity as function of the peak samples and the noise contribution.

As expected from the theoretical analysis at very low levels of the input noise the spectra exhibited large DNL and at very short peaking times some channels did not record counts or recorded a statistically insignificant number of counts. At noise levels $\sigma \geq q$ the DNL becomes small, within the statistical limit of the DNL measurement. This confirms the earlier discussion.

We also operated the ADC at 12-bits (masking the lowest two bits) without adding artificial noise. The only noise was associated with the electronics and the tail pulse generator. The pulse generator produced fixed pulse amplitude at the middle of the ADC range. Three spectra were acquired with total channel lengths of *8k*, *16k* and *32k* channels corresponding to a quantum size reduction by factor of 2, 4 and 8 respectively. The generator tail pulse was shaped by a digital triangular shaper with peak time equal to 128 samples. The full width at half maximum (FWHM) of the spectral peak was measured. The FWHM was 1.73 channels for the *8k* spectrum, 3.36 for the *16k* spectrum and 6.75 for the *32k*. These results along with the DNL measurement indicate that a quantum size reduction is achievable without significant quantization impact on the quality of spectral data.

VII. Conclusion

In this paper we made an attempt to explain and discuss some quantization effects in digital pulse processing in radiation spectroscopy. Some related topics were left out of the scope of this work simply because of space limitation. The differential nonlinearity of the quantizers, the noise bandwidth, artificial dithering, and ADC boundary effects are only a few of the subjects that require consideration in future publications. Nevertheless, the following observations and some design suggestions could be made based on the discussion in this manuscript:

1) Digital pulse height analysis can be viewed as a transformation of the quantized ADC data into quantized spectral data. In this transformation the quantum size of the spectra, q_{chn}, may be smaller than the quantum size of the ADC, q_{ADC}, resulting in more channels than the total number of ADC quantization values.
2) Quantization affects the differential nonlinearity and the overall quality of acquired spectra when the ADC quantum size is reduced to a smaller spectrum quantum size.
3) In order to minimize the quantization effects on the acquired spectra the standard deviation σ of the random noise at the input of the ADC must be greater than the quantization step q of the ADC. As a rule of thumb $\sigma > 3q$ is desirable.
4) Upper noise cutoff frequency at the input of the ADC should be maintained high enough, so that the effective “noise modulation” of the quantization output can be achieved. Analog signal conditioning circuits the highest possible bandwidth should be preferred over high frequency limiting filters such as low-pass networks, ant-aliasing filters, etc. which

reduce both the noise amplitude variance and the noise upper cutoff frequency.

5) The number of ADC samples used to calculate the pulse heights must be sufficiently large to mitigate the quantization effects. Employing ADCs with high conversion speed is essential when digital pulses with short rise time are synthesized.

The above considerations may not be fully applicable when other constraints such as timing, ballistic deficit correction etc. are considered.

The experimental results and the examples in this work cover only a specific class of available techniques and unique hardware implementation. Caution should be exercised in scaling or directly applying the presented results.

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REFERENCES

[1] J.D. Valentine and A.E. Rana, "Centroid and full-width at half maximum uncertainties of histogrammed data with an underlying Gaussian distribution-the moments method", *IEEE Trans Nucl Sci*, vol. 43, pp.2501 - 2508 , 1996.

[2] Glenn F. Knoll, *Radiation Detection and Measurement*, 4th Edition, Wiley, pp.707 – 708, 2010.

[3] Bernard Widrow and Istvan Kollar, *Quantization Noise*, Cambridge University Press, 2008.

[4] G. Ripamonti, A. Castoldi, and E. Gatti, "On the implementation of Multiple Delay Line Filters with digital signal processing techniques", *IEEE Trans. Nucl. Sci.*, vol. NS41, no. 4, pp.1109 - 1115 , 1994.

[5] W.R. Bennett, "Spectra of Quantized Signals," *Bell System Technical Journal*, Vol. 27, July 1948, p. 446-471.

[6] T. Lakatos, "Adaptive digital signal processing for x-ray spectrometry", *NucL Instr. and Meth.,* vol. B47, pp. 307-310, 1990.

[7] V. T. Jordanov and G. F. Knoll "Digital synthesis of pulse shapes in real time for high resolution radiation spectroscopy", *Nucl. Instrum. Methods Phys. Res.*, vol. A 345, p.337 , 1994.