Direct Measurement of Noise Spectra at the 1 nV/√Hz Level

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***Abstract*–In order to confirm that a ultra-low noise power supply was working as designed, we needed to measure its noise spectrum at 15 V at the 1 nV/√Hz level to frequencies below 10 Hz. We accomplished this using the "cross-spectrum" technique. Since this method is not widely known, we describe it briefly, present the preamplifier we developed to make the required measurements, and show spectra from our power supply, whose noise levels approach those of battery supplies.**

I. Introduction

PREAMPLIFIERS in critical applications are are often powered by batteries to achieve the best noise performance because many of the most sensitive front end topologies do not also have excellent power supply noise rejection ratios. However, in developing processing electronics for use with very high resolution superconducting tunnel junction detectors (3 -5 eV FWHM) in arrays of up to 1000 elements, the sheer number of required preamplifiers made this approach impractical and we needed to develop an ultra-low noise power supply whose noise figure would be comparable to that of a battery supply.

A first issue was to determine just what battery noise values are. A literature search showed that this is not a commonly known value, for reasons that will become clear. However, we did manage to locate a single paper, [1] by Boggs and his colleagues at NIST, who used a 2-channel cross correlation technique, coupled with careful, low noise preamplifier design, to measure battery noise as a function of frequency with a noise floor slightly below -200 dBV/Hz (~0.1nV/√Hz). Their results, shown in Fig. 1, show that almost all batteries

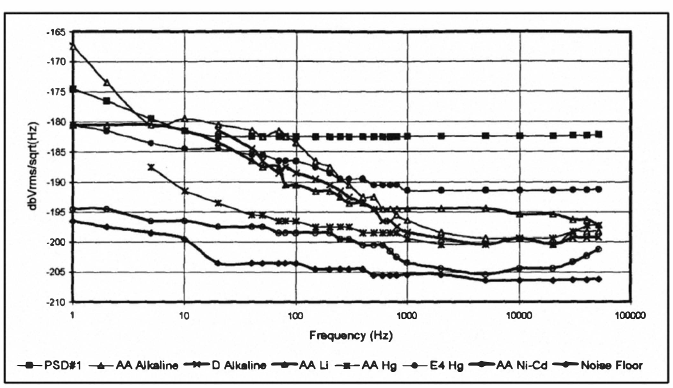

Fig. 1: Noise from various batteries, vs frequency, from Boggs. [1]

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achieve noise levels in the 1 nV/√Hz range, with the better ones being up to a factor of 10 better. This suggested the range of noise performance we would be attempting to achieve..

To avoid over-engineering our power supply, based on these battery numbers, we also performed an independent analysis, following the methods of Fabris *et al.* [2] of the sensitivity of our circuit to power supply noise. This showed that, for the power supply's noise to be negligible, compared to the preamplifier's total noise contribution of < 2 eV, (where the STJ detectors' inherent noise is 8-10 eV at 600 eV) it should should be kept below approximately 4 nV/√Hz, particularly at frequencies above 1 kHz, where our digital filters' bandpasses lie. A major engineering issue, then, was how to measure our power supply's noise spectrum at this level to determine if we were making progress toward our goal. This paper reports on the methods we developed.

II. The Cross-Spectrum Method

A literature survey showed that the preferred approach to this problem is the same one used by Boggs *et al.* [1], namely the so called "cross-spectrum" method. Rubiola and Vernotte (R&V) have presented a particularly clear and comprehensive treatment of the method, which we highly recommend. [3] The following is a greatly abridged version of their presentation.

Consider a device under test (DUT) whose noise, $c(t)$ is simultaneously amplified by two circuits, with gains α and β, that also contribute their own noise terms $a(t)$ and $b(t)$. Their outputs, $x(t)$ and $y(t)$ are then:

$$x(t)=\alpha c(t)+a(t) \text{and } y(t)=\beta c(t)+b(t) \tag{1}$$

We wish to obtain the power spectral density (PSD) S_{cc} of the DUT, typically in the difficult measurement situation where $a(t) \sim b(t) >> c(t)$ (e.g. a 1 nV/√Hz power supply using 10 nV/√Hz amplifiers). Following R&V, we take the product of the Fourier transforms $X(f)$ and $Y(f)$ of $x(t)$ and $y(t)$ and average it over m measurements. Here, for simplicity, we use only the real parts of the transforms, though the result is more general, as explored in R&V. Thus,

$$\begin{aligned}\langle S_{xy}\rangle_m &= \langle X(f)Y(f)\rangle_m = \alpha\beta\langle C(f)C(f)\rangle_m \\ &+\alpha\langle C(f)B(f)\rangle_m + \beta\langle A(f)C(f)\rangle_m + \langle A(f)B(f)\rangle_m\end{aligned} \quad , \tag{2}$$

or

$$\begin{aligned}\langle S_{xy}\rangle_m &= \alpha\beta\langle S_{cc}\rangle_m + \alpha\langle S_{bc}\rangle_m + \beta\langle S_{bc}\rangle_m + \langle S_{ab}\rangle_m \\ &= S_{cc} + O(1/\sqrt{m})\end{aligned} \tag{3}$$

Now, when m is small, the first term is much smaller than the latter three, since $A(f)) \sim B(f) >> C(f)$ because the same is true in the time domain. For larger m values, the average $<S_{xy}>$ converges to the desired PSD S_{cc}. That is, in a well designed system, the *magnitude* of the averaged cross spectrum falls (at any given value of f) until it reaches a floor set by the DUT noise spectrum, as shown in Fig. 2. The *variance* in the

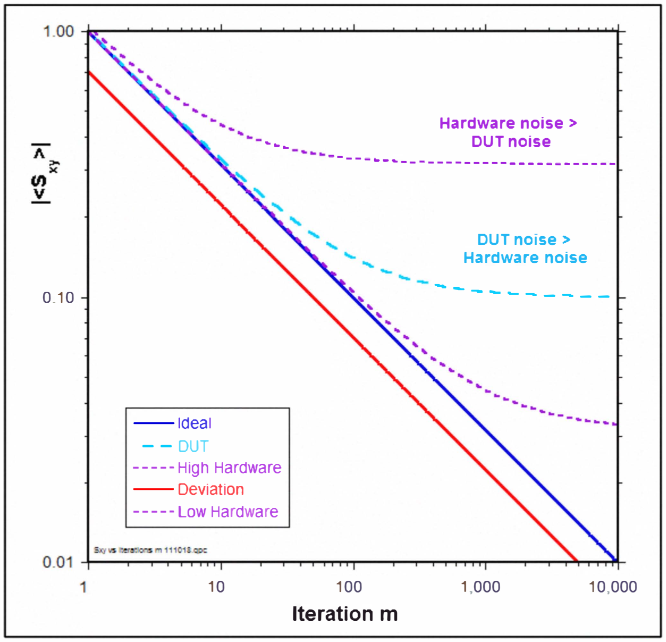


Fig. 2: $<S_{xy}>$ versus number of averages m showing various limits.

values (i.e. the scatter comparing S_{xy} at one f to its neighbors) is smaller (here by the $\sqrt{2}$ and also falls as $\sqrt{m}$ as shown by the "deviation" in Fig. 2.

Examining Figure 2, there are three important limits: the DUT noise floor, the statistical limit, and the hardware limit, where our goal is to reach the DUT noise floor, as shown. The *statistical* limit (shown as "Ideal" in Figure 2) is the course the measurement would follow if the DUT noise were zero (i.e. shorting the two amplifier inputs) and is given by

$$\langle S_{xy}\rangle_m = \sqrt{(\langle S_{aa}\rangle_m \langle S_{bb}\rangle_m)} / \sqrt{m} \quad (4)$$

The *hardware* limit, on the other hand, results from correlated (e.g. ground loop) noise in S_{ca}, S_{bc} and S_{ab}, since this noise will not decay as $\sqrt{m}$ as desired. The hardware limit must clearly be smaller than the DUT limit if we hope to converge to the DUT value. It is worth noting that failing this test at one frequency (e.g. due to strong pickup at 60 and 180 Hz) does not mean that it fails all all frequencies, since they converge independently. Further, even with a very low hardware limit, it is still advisable to make the ratios a/c and b/c as small as possible since convergence only occurs as $1/\sqrt{m}$, which is quite slow. Thus, in Figure 2, DUT noise is about 10% that of the amplifier noise and yet it only starts to emerge from the amplifier noise after more than 100 measurements, and nearly 10,000 measurements are needed for it to converge to its true value. This can be very time consuming. For example, a single spectrum of 5000 points at 500 Hz (e.g. to observe frequencies down to 10 Hz) takes 10 seconds to collect. 10,000 spectra require 28 hours to collect. Achieving a DUT noise level 10 times lower would take 100 times as long, the better part of 1 year, and is clearly not feasible except under the most desperate of circumstances. We will therefore need quite low noise preamplifiers to characterize a 4 nV/√Hz power supply in a reasonable amount of time. We thus select a preamplifier design goal of 8 4 nV/√Hz or less.

III. SIGNAL PREAMPLIFIERS

Our signal preamplifier design specifications are therefore the following: 1) Low noise for measuring supply noise at values significantly below 4 nV/√Hz; 2) DC coupling to a 15V supply so that spectral noise can be measured to 10 Hz or below; 3) enough gain so that 1 nV/√Hz produces a large enough signal to be accurately digitized by a quality 8 bit oscilloscope (i.e. at least 40 mV rms). The oscilloscope

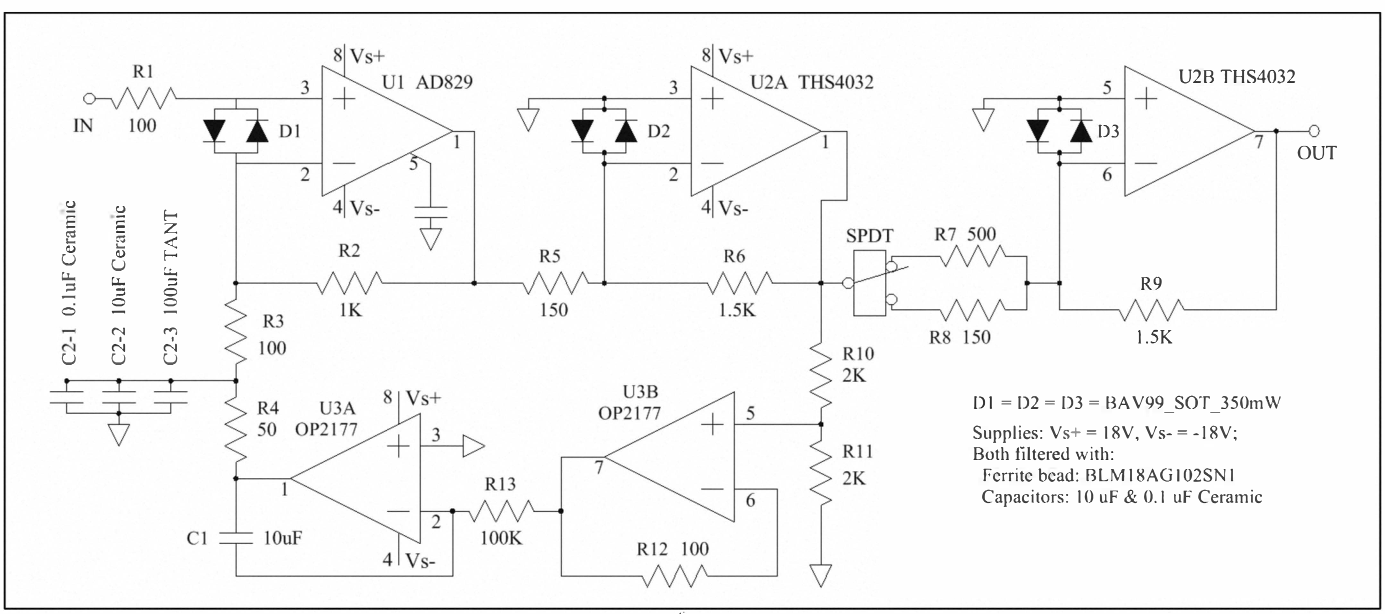


Fig. 3: One of two preamplifier channels for measuring $<S_{xy}>$ from a few nV/√Hz power supply.

amplifier noise does not bother us, it is just another contribution to the channel noise.

Figure 3 shows our preamplifier design. It has two 10X gain stages, followed by a switchable stage with gain of either 3X or 10X. The AD829 front end OpAmp has 1.9 nV/√Hz noise. Input referenced noise is about 4 nV/√Hz. With 1000X gain and a 20 MHz scope bandwidth, this give about 16 mV rms output signal, which is adequate with the scope set between 20 and 40 mV/division input. Each amplifier has a pair of diodes across its inputs to limit the differential input voltage until the circuit stabilizes when connected to a non-zero voltage source.

The interesting part of this circuit is the feedback loop using OpAmps U3A and U3B. This circuit samples the output at the end of the second gain stage, buffers and integrates it, and feeds it back to the negative input of the input OpAmp U1. With a time constant (1 sec) set by the integrator (R13 & C1), this circuit matches the DC component of the input signal so that only the small AC noise (1-4 nV/√Hz) riding on the large (15V) component is finally amplified 1000-fold. The output of U3A is heavily filtered (R4 & C2) to keep its noise out of the circuit's input.

IV. Spectral Results

We collected data by attaching the two preamplifier outputs to input channels 1 & 2 of a Tektronix DPO7254 oscilloscope and using its Math capability to compute:

$$Math(4) = Avg(\sqrt{(SpectralReal(Ch1)*SpectralReal(Ch2))}) \quad (5)$$

as each pair of traces was captured. Traces were captured in HiRes mode, which averages samples between points to suppress Nyquist aliasing. Fig. 4 shows <*Sxy*> traces captured between 0 and 250 Hz with the two channel inputs tied to the same 62Ω resistor as a nominal 1 nV/√(Hz) white noise generator. Because the noise levels of the source and amplifiers are not very different, the traces do not move much

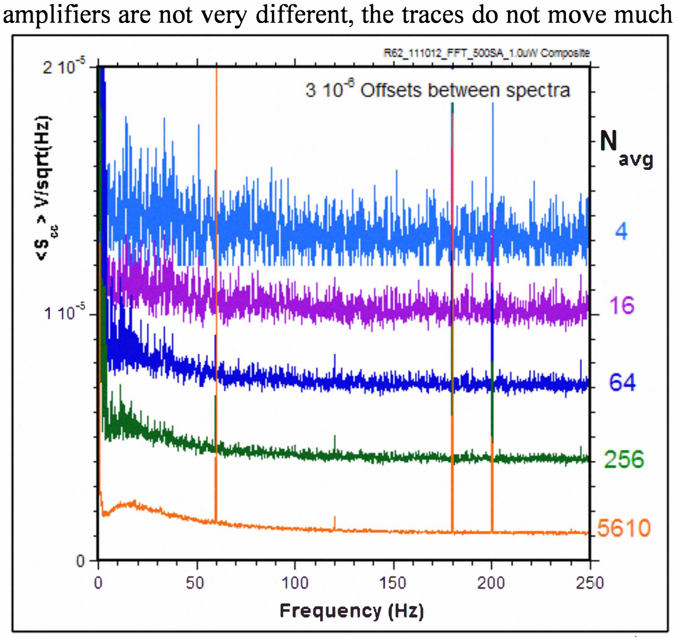
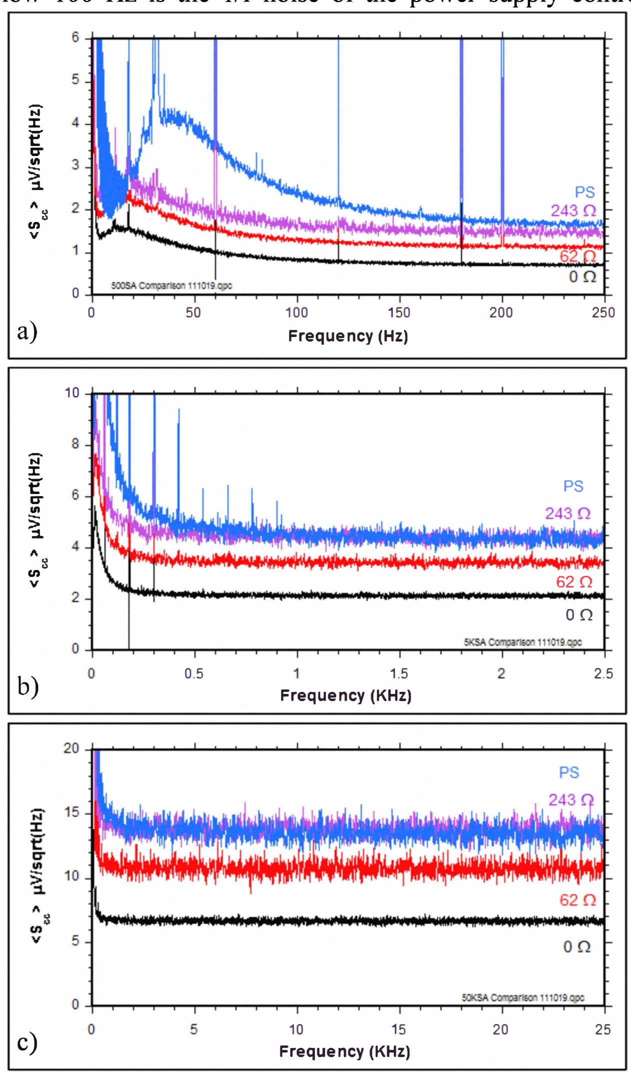


Fig. 4: <*Sxy*> vs *f* for several averages *m*, using a 62Ω resistor as a 1 nV/√Hz noise source.

as the number of averages ***m*** increases and only reduce their variance. It is clear that quite a number of traces must be averaged to resolve <*Sxy*> in full detail. The final curve took approximately 16 hours to collect at 10 sec/trace. The spikes at 60, 180 & 200 Hz are common mode noise picked up by both preamplifiers.

Figures 4a to 4f show <*Sxy*> data from 0 to 25 MHz that compare our low noise power supply to three resistor noise generators having nominal noise densities of 0, 1 and 2 nV/√(Hz). The design is highly successful in that, from about 500 Hz to 2.5 MHz, its noise is approximately 2 nV/√(Hz), the same as the 243Ω resistor. Recalling, from Eqn. 3 that our measurement of <*Scc*> contains the two preamplifier gains α and β, it is probably that the power supply noise is still good above 2.5 MHz but that the amplifier circuit gain has a broad slight resonance similar to the one shown when the 243Ω resistor is measured. At the low frequency end the amplifiers show some residual pickup at 60, 180 and 200 Hz which appears in all the traces. The power supply peaks below 50 Hz are actually mechanical resonances, probably from re-work wires on the PC board and change depending upon how the board is damped. The peak in the power supply noise below 100 Hz is the 1/f noise of the power supply control



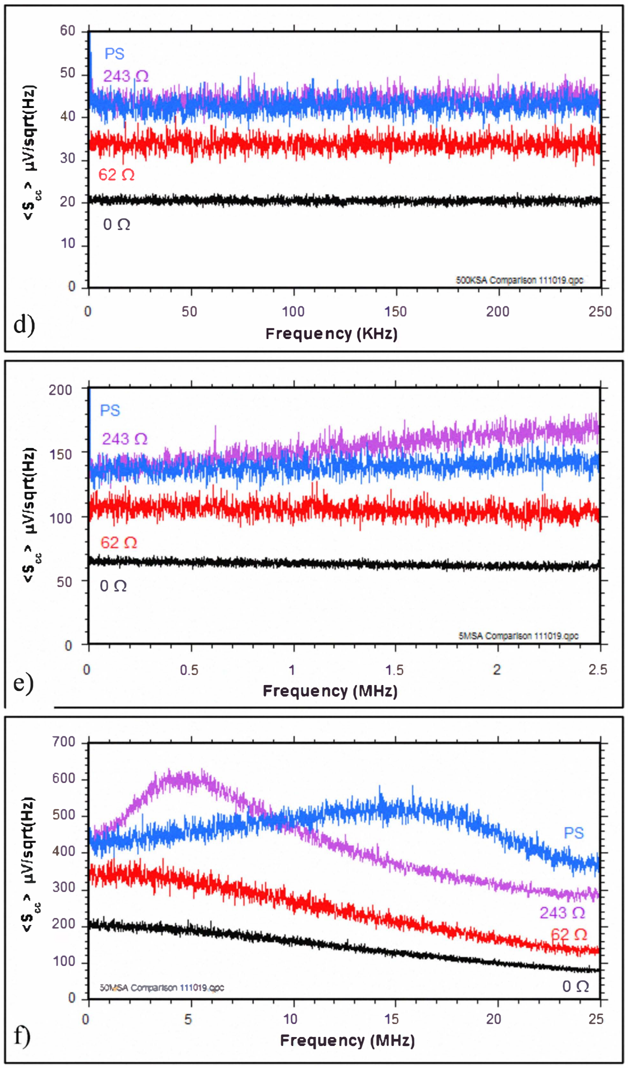


Fig. 5: Comparing <Sxy> from our power supply (PS) to <Sxy> from 0, 62 and 243Ω resistors over 6 frequency ranges from a) 250 Hz to f) 25 MHz.

OpAmp, cut off at low frequency by the amplifiers' nulling circuits. These examples provide simple demonstrations of the power of this technique and its utility in identifying noise sources when working with ultra-low noise circuits of any kind.

V. Conclusions

We have reviewed the cross-spectrum method for measuring noise spectra from circuits whose noise is less than that of available measurement instruments and presented a design for a preamplifier that allows one to make noise measurements in the 1 nV/√Hz range from 0 to 2.5 MHz on a 15V power supply. Comparisons to the noise from resistor noise sources showed that the power supply attained its design goal of less than 4 nV/√Hz.

References

[1] C. K. Boggs, A. D. Doak and F. L. Walls, "Measurement of voltage noise in chemical batteries", 1995 IEEE International Frequency *Control Symposium,* 1995, pp. 367-373.

[2] L. Fabris, N. W. Madden and H. Yaver, "*A* fast, compact solution for low noise charge preamplifiers", NIM A 424 (1999) pp. 545-551.

[3] E. Rubiola and F. Vernotte, "The cross-spectrum experimental method", is available as a pdf file at: http://arxiv.org/PS_cache/arxiv/pdf/1003/1003.0113v1.pdf.