Recursive Algorithms for Real-Time Digital $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ Pulse Shaping

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***Abstract*—This paper reports on recursive algorithms for real-time implementation of $\mathrm{CR}–(\mathrm{RC})^{\mathrm{n}}$ filters in digital nuclear spectroscopy systems. The algorithms are derived by calculating the Z-transfer function of the filters for filter orders up to $\mathrm{n}=4$. The performances of the filters are compared with the performance of the conventional digital trapezoidal filter using a noise generator which separately generates pure series, 1/f and parallel noise. The results of our study enable one to select the optimum digital filter for different noise and rate conditions.**

***Index Terms*—Digital signal processing, gamma spectroscopy, semi-Gaussian filter.**

I. Introduction

In recent years, the performance of nuclear spectroscopy systems has been considerably improved by replacing the conventional analogue electronics modules by modern digital systems. In digital systems, the detector signals are digitized directly after the preamplifier stage and the pulse processing operations such as baseline correction, pulse shaping and pile-up correction are carried out on a digital hardware using dedicated pulse processing algorithms. In this way, an important role is played by the pulse shaping algorithm. The function of a pulse shaper is to shape the signals to optimize spectrometer performance which might involve a compromise between several parameters such as signal-to-noise performance, high rate operation and insensitivity to rise-time fluctuations in the detector signal. The digital filter should also be implementable with minimum computational effort in order to be suitable for real-time applications. In general, a good compromise between the aforementioned parameters is made by the digital trapezoidal shaper [1], [2] and this filter is the common choice in digital spectroscopy systems. However, there are other alternative pulse filters which might be advantageous under some noise, charge collection and rate circumstances.

One of the pulse shaping methods, which has been widely used in the analogue nuclear spectroscopy systems, is the $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ shaping. This filter is based on a simple network of one differentiator followed by a series of n integrators in a suitable cascade arrangement. The number of integrators is called the order of the filter. In the present work, we derive recursive algorithms for digital implementation of $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ pulse filters for filter orders up to four. The performances of the filters against pure series, 1/f and parallel noise are compared to that of the digital trapezoidal filter. The results of our study reveal that a digital $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filter performs considerably better than a digital trapezoidal filter, when the parallel noise is the dominant noise.

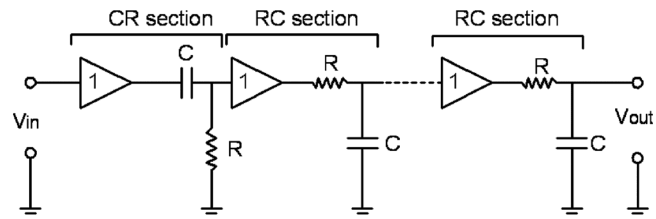

Fig. 1. A $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ shaping network.

II. Digital Filter Design

Fig. 1 shows a $\mathrm{CR}–(\mathrm{RC})^{\mathrm{n}}$ filter. The filter is realized by a CR differentiator which is followed by a number of RC integrators. Theoretically, a Gaussian pulse shaper is realized when an infinite number of integration stages are used. In practice, normally maximum four stages of integration $(\mathrm{n}=4)$ are used and the resulting $\mathrm{CR}-(\mathrm{RC})^{4}$ filter is called a semi-Gaussian filter, as its output is only an approximation of the Gaussian shape. The impracticality of using more than four integration stages is due to the fact that further integration stages lead to very small improvement to the overall noise performance of the filter, while a considerable delay is caused by each stage and it also increases the number of components and complexity of the circuit. If the differentiation and n integration time constants are all the same value τ, the transfer function of the corresponding circuit is described by:

$$H(s)=\frac{s\tau}{(1+s\tau)^{n+1}} \tag{1}$$

The step response of a $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filter is given by u(t) as:

$$u(t)=\frac{1}{n!}\cdot\left(\frac{t}{\tau}\right)^{n}\cdot e^{-(t/\tau)} \tag{2}$$

and the transient gain K, defined to be the ratio of the peak output voltage to the step input voltage, is given by:

$$K=\frac{n^{n}}{n!}\cdot e^{-n} \tag{3}$$

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The $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters are infinite impulse response filters and their recursive forms can be calculated directly from the Z-transform of the impulse response function. The impulse response function h(t) is a differentiation of the step response function u(t). If one assumes the input signals to the digital filter are sampled with a sampling interval of T, the Z-transform of the impulse response function H(z) will be:

$$H(z) = \sum_{m=0}^{\infty} h(mT) z^{-m} \tag{4}$$

where m is an integer and z is, in general, a complex number. We directly calculated (4) for $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters associated with n values from 1 to 4, leading to the following Z-transfer functions [3]:

$$H(z)_{CR-RC} = \frac{z^2 - z\alpha(1+aT)}{(z-\alpha)^2}, \tag{5}$$

$$H(z)_{CR-(RC)^2} = \frac{z^2 T\alpha(2-aT) - zT\alpha^2(2+aT)}{2(z-\alpha)^3}, \tag{6}$$

$$H(z)_{CR-(RC)^3} = \frac{z^3 T^2 \alpha(3-aT) - 4az^2T^3\alpha^2 - zT^2\alpha^3(3+aT)}{6(z-\alpha)^4}, \tag{7}$$

$$H(z)_{CR-(RC)^4} = \frac{z^4 \alpha T^3(4-aT) + z^3\alpha^2T^3(12-11aT) +}{24(z-\alpha)^5} \frac{z^2\alpha^3T^3(-12-11aT) + z\alpha^4T^3(-4-aT)}{24(z-\alpha)^5}, \tag{8}$$

where $\mathrm{a} = 1/\tau$ and $\alpha = \exp(-\mathrm{T}/\tau)$. Having obtained the Z-transfer functions and by using the general relation:

$$\mathrm{Y(z)} = \mathrm{H(z)X(z)}, \tag{9}$$

where Y(z) and X(z) are, respectively, the Z-transforms of the output and input signals, the corresponding difference equations (recursive algorithms) of the filters are calculated by taking the inverse Z-transform of (9). The calculated recursive formulas are summarized in Table I. The formulas indicate that the amount of computation increases with the order of filter, as, for instance, each output sample of a CR-RC filter is computed by performing four additions and three multiplications, while in the case of a $\mathrm{CR}-(\mathrm{RC})^4$ filter, nine additions and nine multiplications are required to compute each output sample.

Fig. 2 shows the response of the filters to a sampled input signal from a nuclear pulse generator. The peaking time of a $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filter is equal to the product of the filter's order (n) and the time constant ($\tau = \mathrm{RC}$). In Fig. 2, the same peaking time is set for filters of different orders by changing the time constant of the filters. In a view of pulse pile-up, it is important that the filter's output returns to the zero level as quickly as possible in order to not affect the amplitude of the successive events. In the case of $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters, the width of output decreases with the filter order so that the duration of a semi-Gaussian filter is considerably shorter than that of a CR-RC filter. However, even for a semi-Gaussian filter, the signal continues for a considerable time after the peaking time of the signal, while this part of the signal carries no information on the pulse amplitude. This is a major drawback of semi-Gaussian shaping in analogue domain, leading to a considerable pulse pile-up at high rates. However, in the digital domain, it is possible to limit the digital signal processing to a finite time window [4] which is only long enough to contain the pulse maximum and, therefore, minimizing the time spacing between the events. This enables a considerable improvement in the rate capability of the filters, as compared to that of the standard analogue $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ pulse filters. Another useful feature, extractable from the shape of the signals in Fig. 2, is that the CR-RC shaping exhibits the minimum rate of curvature at the peak of the step response. This gives the filter a higher

TABLE I
DIGITAL RECURSIVE ALGORITHMS OF $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ FILTERS FOR FILTER ORDERS OF 1 TO 4

FILTER TYPE	RECURSIVE ALGORITHM
CR-RC	$y[n]_{CR-RC} = 2\alpha y[n-1] - \alpha^2 y[n-2] + x[n] - \alpha(1+aT)x[n-1]$
CR-(RC)^2	$y[n]_{CR-(RC)^2} = 3\alpha y[n-1] - 3\alpha^2 y[n-2] + \alpha^3 y[n-3] + T\alpha(1-\frac{aT}{2})x[n-1] - T\alpha^2(1+\frac{aT}{2})x[n-2]$
CR-(RC)^3	$y[n]_{CR-(RC)^3} = 4\alpha y[n-1] - 6\alpha^2 y[n-2] + 4\alpha^3 y[n-3] - \alpha^4 y[i-4] + T^2\alpha(\frac{1}{2} - \frac{aT}{6}aT)x[n-1] - (\frac{2aT^3\alpha^2}{3})x[n-2] - T^2\alpha^3(\frac{1}{2} + \frac{aT}{6})x[n-3]$
CR-(RC)^4	$y[n]_{CR-(RC)^4} = 5\alpha y[n-1] - 10\alpha^2 y[n-2] + 10\alpha^3 y[n-3] - 5\alpha^4 y[n-4] + \alpha^5 y[n-5] + (1/24)(-aT^4\alpha + 4T^3\alpha) x[n-1] + (1/24)(-11aT^4\alpha^2 + 12T^3\alpha^2)x[n-2] + (1/24)(-12T^3\alpha^3 - 11aT^4\alpha^3)x[n-3] + (1/24)(-aT^4\alpha^4 - 4T^3\alpha^4)x[n-4]$

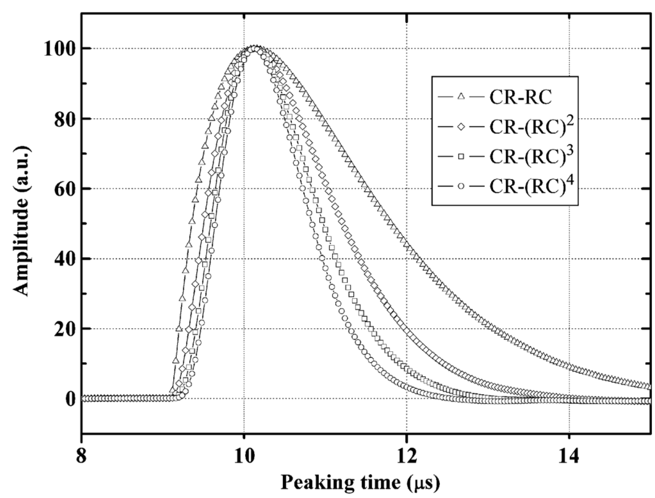

Fig. 2. The output signals calculated by means of the algorithms of Table I. The input is a signal from a pulse generator, sampled at 75 MHz with 14 bit resolution. The minimum rate of curvature at the peak of the outputs is seen for the CR-RC filter.

immunity to ballistic deficit [5], which is a source of degradation in the energy resolution of detectors with large variations in the rise-time of signals.

III. Noise Performance Simulation

The main sources of electronic noise in semiconductor detectors are the parallel noise due to detector leakage current, the series noise generated in the input FET of the preamplifier and the 1/f noise in the detector-preamplifier assembly. We compared the performance of $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters of different orders against each of these sources of noise. For this purpose, a program in MATLB environment was written which separately generates pure series noise, 1/f noise and parallel noise. Then, the outputs of the noise generator were added to a noiseless step pulse and twenty thousands of noisy pulses were produced for each type of noise. The noisy pulses were processed using algorithms of Table I and the full-width-at-half-maximum (FWHM) of the pulse height spectra were calculated using a Gaussian fit. Fig. 3 shows the results of FWHM calculations as a function of the peaking time of the filters. Fig. 3(a) shows that the noise performance of the filters against the series noise degrades with the filter's order so that the CR-RC filter has the best performance against the series noise. Fig. 3(b) shows that the performance of the digital $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters against the 1/f noise improves very weakly with the peaking time. Moreover, a slight improvement in the performance of the filters with the filter's order is observed so that the best performance against the 1/f noise is achieved with the $\mathrm{CR}-(\mathrm{RC})^4$ filter. In regard to the parallel noise, Fig. 3(c) indicates that the noise performance improves with the filter's order and the best performance is achieved by the $\mathrm{CR}-(\mathrm{RC})^4$ filter.

From the results of the Fig. 3, one can conclude that among the derived algorithms either CR-RC filter or $\mathrm{CR}-(\mathrm{RC})^4$ filter shows the best performance in regard to different sources of noise. In the next step, the performance of CR-RC and $\mathrm{CR}-(\mathrm{RC})^4$ filters were compared against the performance of trapezoidal filter, as the most widely used digital filter. The trapezoidal filter was realized by using the algorithm reported in [1]. The time scale of trapezoidal filter is determined by the filter's peaking time and the length of flat-top region. Fig. 4 shows the results of noise performance comparison of CR-RC, $\mathrm{CR}-(\mathrm{RC})^4$ and trapezoidal filters of different flat-top length as a function of the peaking time of the filters. The results show that in regard to the series noise, trapezoidal filter behaves considerably better than CR-RC filter which has the best performance among the $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters. This is opposite to the theoretical noise calculations in [6] which reports the same parallel noise indices for trapezoidal and semi-Gaussian filters. It can also be inferred that, despite the results of previously calculated noise indices for trapezoidal filter [6] in which the series noise index is independent of the length of flat-top region, the performance of trapezoidal filter slightly improves with the length of flat-top region and only at longer peaking times the noise performance tends to be independent from the length of flat-top region. Fig. 4(b) shows that, in general, the performance of trapezoidal filter against the 1/f noise is slightly better than that of semi-Gaussian filter. However, the performance of the trapezoidal filter degrades with the length of flat-top region so

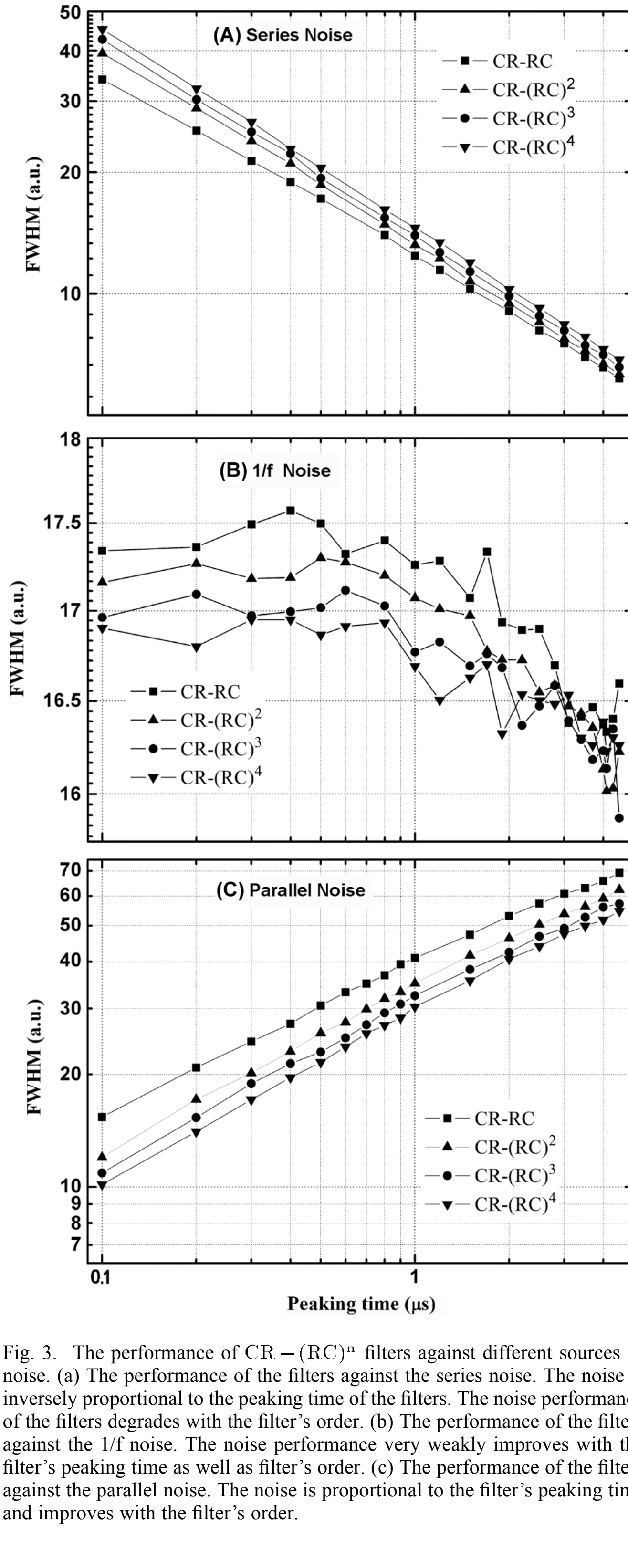

Fig. 3. The performance of $\mathrm{CR}-(\mathrm{RC})^{\mathrm{n}}$ filters against different sources of noise. (a) The performance of the filters against the series noise. The noise is inversely proportional to the peaking time of the filters. The noise performance of the filters degrades with the filter's order. (b) The performance of the filters against the 1/f noise. The noise performance very weakly improves with the filter's peaking time as well as filter's order. (c) The performance of the filters against the parallel noise. The noise is proportional to the filter's peaking time and improves with the filter's order.

that the best performance is achieved with the triangular filter (trapezoidal filter with zero flat-top length). Fig. 4(c) shows that the performance of semi-Gaussian filter against the parallel noise is considerably better than that of triangular filter and the noise performance of trapezoidal filter degrades with the length of flat-top region. This is different from the results of theoretical

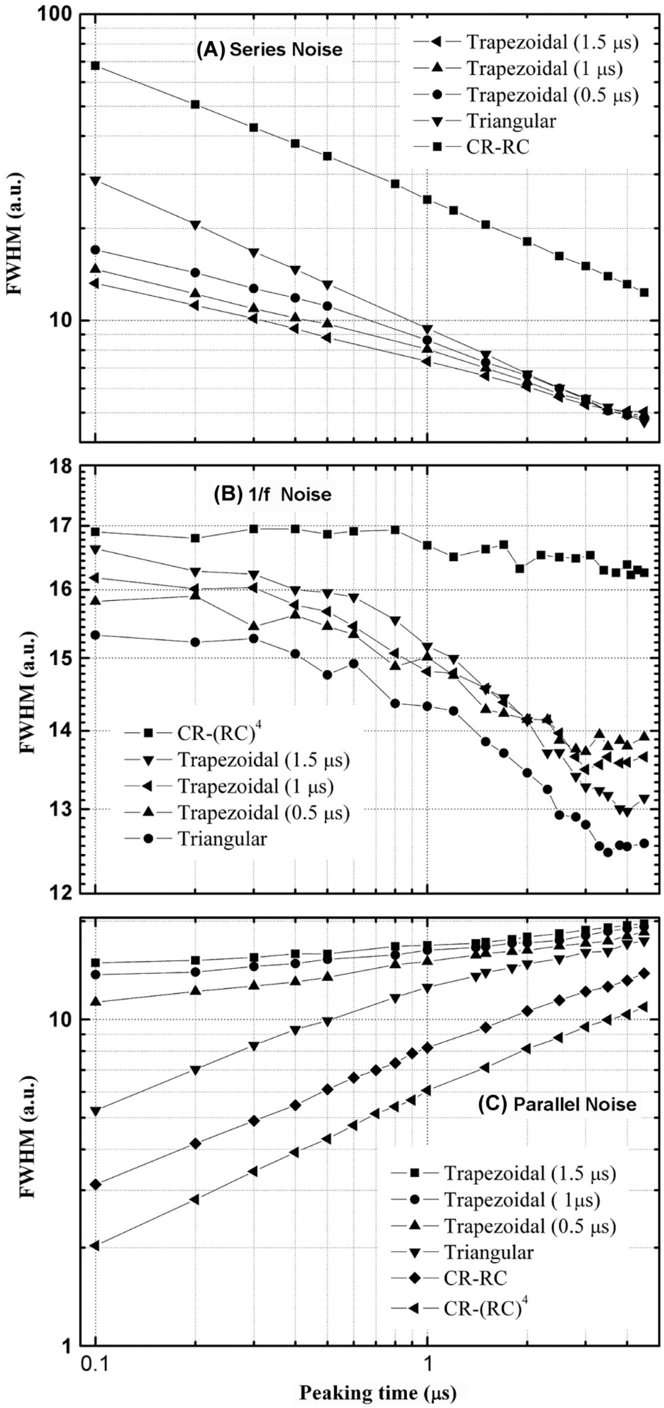

Fig. 4. (a) The performance of CR-RC and trapezoidal filters of different length of flat-top region against the series noise. The length of flat-top region is given within the brackets. The results demonstrate that, in regard to series noise, the best result is obtained with the trapezoidal filter (b) The performance of trapezoidal filter and semi-Gaussian filter against the 1/f noise. The best performance is achieved with triangular shaping. (c) The noise performance of CR-RC, semi-Gaussian and trapezoidal filters of different flat-top length against the parallel noise. Both the CR-RC and semi-Gaussian filters behave considerably better than the trapezoidal filters.

noise index calculation [6] which reports for triangular shaping a better parallel noise index than that for semi-Gaussian shaping. Since the parallel noise is mainly caused by the leakage current of detectors, for detectors with large leakage current, i.e. room temperature detectors, a semi-Gaussian filter is a much more effective choice than trapezoidal filter. Fig. 4(c) indicates that, in regard to parallel noise, even CR-RC filter has a better performance than trapezoidal filter. It was also described in Section II that CR-RC filter has a high resilience against ballistic deficit. These properties of CR-RC filter combined with the possibility of minimizing the time spacing between the events in digital domain, make the digital CR-RC filter a good choice for pulse processing of semiconductor detectors at high rates, where a filter with short width and minimum sensitivity to ballistic deficit is required. The performance of such filter is somewhat similar to the analogue CR-RC switched filter [7], which eliminates the long fall-time of the filter by returning to zero-level after the pulse peaking time.

IV. Conclusion

In this paper, recursive pulse processing algorithms were developed for real-time implementation of $\mathrm{CR}-(\mathrm{RC})^n$ pulse filters for filter orders up to four. The noise performance of the filters was analyzed and compared against the performance of the conventional digital trapezoidal filter using pure sources of series, parallel and 1/f noise. The results of our study reveal that (i) in regard to parallel noise, opposite to the results of the previous noise index calculations, a $\mathrm{CR}-(\mathrm{RC})^4$ filter has a better performance than a trapezoidal filter. (ii) Under the noise condition that the series noise is the dominant noise, the trapezoidal filter is the best choice. (iii) The best performance against the 1/f noise is achieved using the trapezoidal filter. (iv) A digital CR-RC filter is a good choice in the high rate conditions that require a filter with high resilience to ballistic deficit and parallel noise.

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