Investigation of Different Wavelets for Pulse Shape Discrimination of LSO and LuYAP Scintillators in Positron Emission Tomography

A. A. Arafa, H. I. Saleh, M. Ashour, A. Salem

***Abstract*—A way to get the depth of interaction (DOI) information in a positron emission tomography (PET) scanner is the use of phoswich detectors. The layer of interaction is identified from the pulse shape of the corresponding scintillator material. In this work, wavelets based pulse shape discrimination (PSD) were investigated in order to find a best practical wavelets family of distinguishing two different pulses types recorded from LSO and LuYAP crystals. The wavelets based PSD gain high performance ranges from 99.75 to 99.26. On the other hand, the best compromise between performance and Discrete wavelet transform (DWT) decomposition level and execution time turned out that Dubechies 6 (db6) gives 99.63% successful discrimination rate at level 1 and consuming 12.828 sec over 10 000 pulses.**

***Index Terms*—PSD, DWT, PET, DOI , Crystal Identification**

I. Introduction

The Digital positron emission tomography (PET) scanner architectures offer high flexibility and reliability over analog ones [1]. The use of a stack of two different scintillation materials optically coupled to a single photomultiplier tube (PMT), known as phoswich detector (shown in Fig. 1) [2]. Recent papers are dealing with different approaches to gain the DOI information. Pulse shape discrimination (PSD) methods use temporal information of pulses. Although they are quite easy to implement, they are also fairly sensitive to noise and, mostly limited to specific crystal pairs with rather different characteristics. On the other hand, performing wavelet decomposition on the real-time signals is an alternative. But to start studying wavelets, one of the many questions "How does one decide which wavelet algorithm to use?" There is no absolute answer to this question. The choice of the wavelet algorithm depends on the application. So the wavelets different family is investigated under the many parameters. These parameters are the performance, decomposition level, and execution time which indicate the complexity of the calculating. In the following section, the system setup and the pulse recording are described. In section III, a brief discussion on discrete wavelets transform and the used family is introduced. Section IV describes the procedure and feature extraction method. The result is shown in section V. Finally, section VI gives the discussions and conclusions.

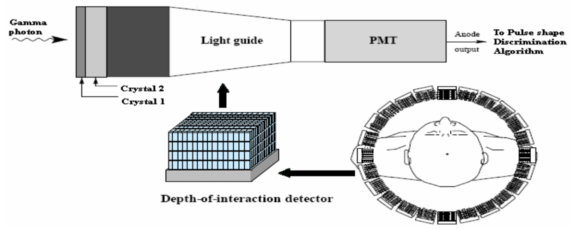

Fig.1. DOI (Phoswich) Detector[2].

II. Pulse Recording

As a base for analyzing the properties of the light output a set of 10 000 pulses was recorded for two crystals (LSO, LuYAP). The crystals are polished and of size 2X2X10 mm. They were positioned individually on a photomultiplier tube. Scintillation pulses were recorded by means of an acquisition board described in [1]. The interesting pulse signal is picked up at the last dynode of the PMT. The signal is amplified and has to pass a low-pass filter in order to satisfy the Nyquist theorem prior to digitization. For this purpose two types of passive second order LCR filter with a cut-off frequency: (1) at 3.6 MHz (2) at 10 MHz and an attenuation of 3 dB is used. Finally, each pulse is represented by 16 samples recorded at a sampling frequency of 40 MHz, thus covering a time window of 400 ns.

In order to get an impression of the different pulse characteristics, Fig. 2, and Fig. 3 show average of the recorded and normalized data at different condition, respectively. Notice Fig 2, It can clearly be seen that LuYAP has a significantly lower light output (lower energy) than LSO. While Fig 3, clearly shows that LuYAP has a significantly slow decay than LSO.

In Fig 3, each pulse is normalized to the peak value of that pulse; so the peak value of each pulse is reduced to 1 and resulted pulse is described by:

$$\hat{x}_i = \frac{x_i}{x_{\max}} \tag{1}$$

For the purpose of the experiment study; there are four groups of samples for each of the two scintillation crystals (LSO, LuYAP). The first two are the normalized 3 MHz and

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10 MHz filtered data. The second two are un-normalized 3 MHz and 10 MHz filtered data.

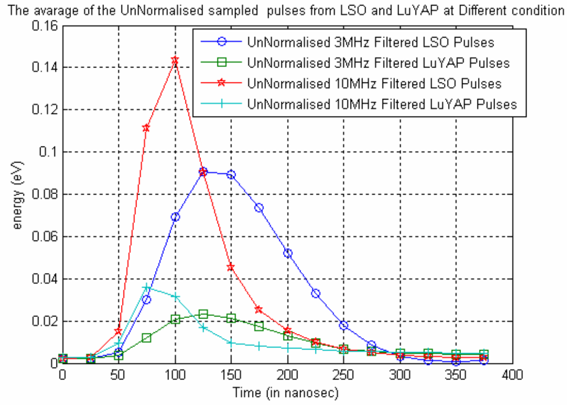


Fig.2. Un normalized Filtered (3 MHz, 10 MHz) and sampled pulses at 40 MHz from LSO, LuYAP each averaged over all recorded events

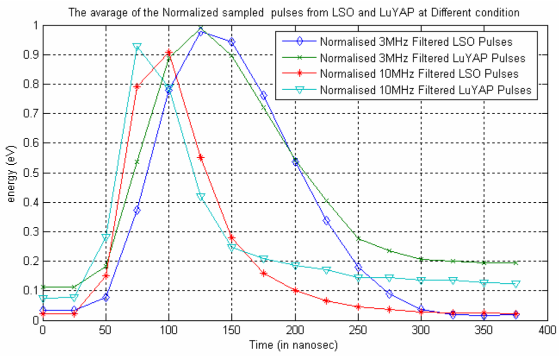


Fig.3. Normalized Filtered (3 MHz, 10 MHz) and sampled pulses at 40 MHz from LSO, LuYAP each averaged over all recorded events

III. Discreet Wavelet Transform

The discrete wavelets transform (DWT) [3] algorithm is desirable which can handle more pulses in the same time and therefore reduce the required calculation power. DWT involves decomposing signals into its constituent parts. DWT offers advantages over other analysis methods for signals containing sharp transients (i.e. like nuclear pulses issued from PET detectors) and discontinuities. The analysis signal used in DWT is known as a wavelet, and is described by

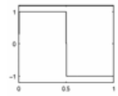
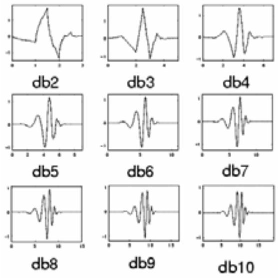
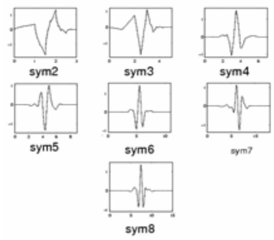
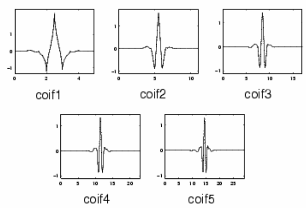
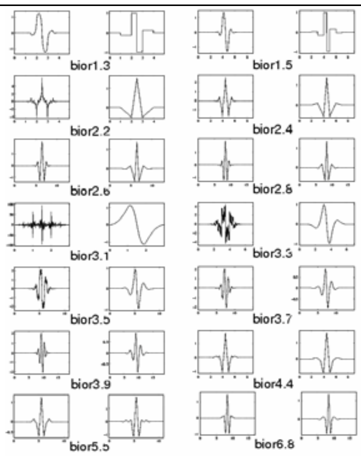
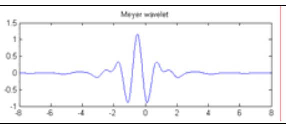
$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), a \in R^{+}, b \in R. \quad (2)$$

Where: a represents the scale parameter, and b the translation parameter. The wavelet is repeatedly shifted, scaled, and convolved with the analyzed signal to extract temporal and spectral component.

Although good temporal and frequency resolution are obtained with Continues Wavelet Transform (CWT), it requires high processing time, memory usage, and, moreover, generates a huge amount of redundancy. For these reasons, the DWT is preferred for computation. The DWT minimizes the redundancy while retaining enough information for accurate analysis and synthesis of the signal [5].

Many wavelets have been proposed in literature [3], [4]. However, some are more popular than other for their characteristic or their performance. Table 1 depicts the different wavelet function which used in this investigation.

Table 1lists the mother wavelet for diffeerent used wavelets families

Wavelet Family Name	**Shape**
Haar wavelet	
Daubechies wavelets	
Symlets	
Coiflets	
Biorthogonal wavelets	
Discrete approximation of Meyer wavelet	

Given a signal x of length n, the DWT consists of log2n stages at most. Starting from x, the first step produces two sets of coefficients: approximation coefficients A1, and detail coefficients D1 as shown in Fig 4. If the length of each wavelet filter is equal to 2N. If n = length (x) the coefficients A1 and D1 are of length floor((n-1)/2)+N The next step splits the approximation coefficients A1 in two parts using the same scheme, replacing x by A1 and producing A2 and D2, and so on. Table 2 shows the applying of the previous analysis on the used wavelet family and calculating the execution time over 10 000 pulses for each decomposition level using MATLAB.

Table 2 Summary of used WVT in the investigation analysis and their coefficient count and execution time at each level of decomposing

WVT family	coef count at Level 1	Exec. time of level 1 (sec)	coef count at Level 2	Exec. time of level 2 (sec)	coef count at Level 3	Exec. time of level 3 (sec)
haar	8	9.031	4	5.734	2	5.047
db2	9	9.125	6	7.797	4	6.453
db3	10	10.047	7	8.203	6	7.937
db4	11	11.094	9	9.515	8	9.563
db5	12	11.922	10	10.703	9	9.719
db6	13	12.828	12	12.187	11	11.453
db7	14	13.593	13	13.093	13	14.438
db8	15	14.735	15	15.125	15	14.750
sym3	10	9.875	7	8.000	6	8.156
sym5	12	11.718	10	10.454	9	9.531
sym6	13	12.500	12	11.953	11	11.063
sym7	14	13.391	13	12.719	13	13.375
sym8	15	15.656	15	15.672	15	15.234
bior1.3	10	12.235	7	11.516	6	10.000
bior1.5	12	14.093	10	12.235	9	11.328
bior3.1	9	12.313	6	10.781	4	9.797
bior3.3	11	14.609	9	12.453	8	12.563
bior3.5	13	15.563	12	14.766	11	14.172
bior3.7	15	17.562	15	17.656	15	17.563
bior3.9	17	18.969	18	19.609	18	19.610
bior5.5	13	16.297	12	15.625	11	15.125
bior2.2	10	12.125	7	10.125	6	9.547
bior2.4	12	13.656	10	12.110	9	11.484
bior2.6	14	15.531	13	14.515	13	14.453
bior2.8	16	17.578	16	17.062	16	17.156
bior4.4	12	15.250	10	13.953	9	13.297
bior6.8	16	18.953	16	18.985	16	18.953
coif1	10	9.765	7	7.812	6	7.265
coif2	13	12.234	12	11.579	11	10.625
coif3	16	15.078	16	14.594	16	14.657
coif4	19	16.687	21	18.047	22	18.859
coif5	22	19.000	25	20.844	27	22.234
dmey	58	53.938	79	73.516	90	83.312

IV. PROCEDURE

A. The discrete wavelet transform Stage

Although the DWT is a very powerful method used in the compression field, it can be used for crystal identification by decomposing the signal into its DWT coefficients.

The DWT analysis operation is similar to a simple convolution and its implementation follows a recursive filter scheme known as a two-channel subband coder. The filter design, used at each decomposition stage, will be repeated for different wavelet functions listed in table 2. The DWT generates the approximation coefficients $A_j[k]$ and detail coefficients $D_j[k]$ for the decomposition of the signal $x_i(t)$ into its scaling function $\varphi(t)$ at scale j and wavelet function $\psi(t)$ at scale j.

The algorithm scheme of discrete wavelet decomposition is shown in Fig. 4. The wavelet approximation coefficients at levels 1, 2, 3 distributions from 100 samples are reported in Fig. 5, 6, 7 respectively where crystal identification performance achieves nearly a perfect score.

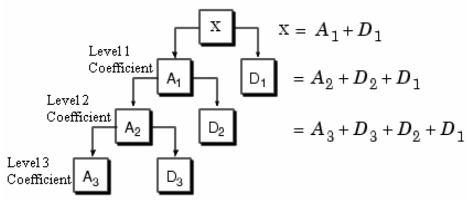


Fig. 4 The Discrete Wavelet decomposition tree

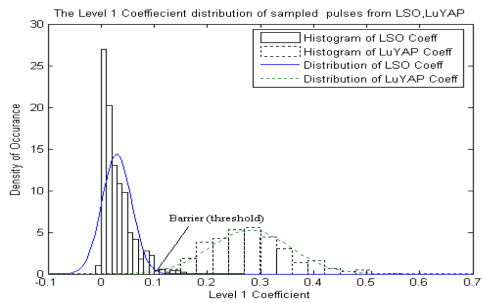


Fig.5 the Discrete Wavelet Level 1 approximation coefficient distribution of LSO and LuYAP

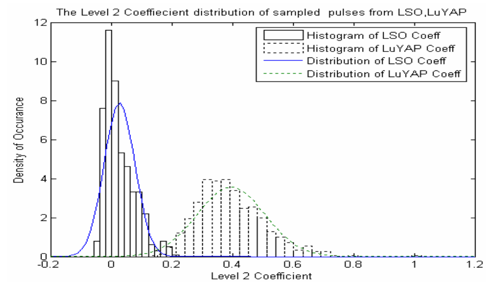


Fig.6 the Discrete Wavelet Level 2 approximation coefficient distribution of LSO and LuYAP

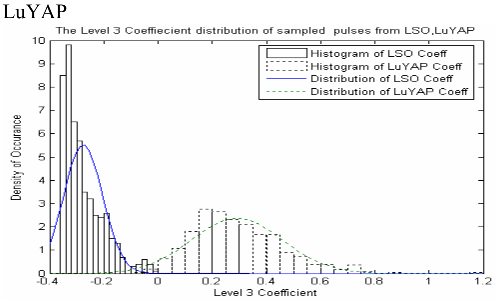


Fig.7 the Discrete Wavelet Level 3 approximation coefficient distribution of LSO and LuYAP

B. Feature Extraction

By analyzing the different DWT coefficients at different level, the best one can be deduced to be used for a clear separation of LSO from the LuYAP materials. This can be mad by three steps.

First: Using the hypothesis test and notice the histogram of the different coefficient, it can be assumed that the coefficients data fit normal distribution.

Second: A threshold is calculated (shown in Fig. 5) using 1% and 10% of the recorded pulses (i.e. 50 and 500 for each crystal). For the purpose of investigation of the best threshold value, two methods are tested:

1. *Statistical Equation method.*

   The mean (μ_{LSO}, μ_{LuYAP}) and standard deviation (σ_{LSO}, σ_{LuYAP}) for each DWT coefficient at the three level of decomposition are calculated. There is a different threshold value at each coefficient at each level can given by the following equation

$$Threshold = \frac{\sigma_{LuYAP}\,\mu_{LSO} + \sigma_{LSO}\,\mu_{LuYAP}}{\sigma_{LSO} + \sigma_{LuYAP}} \qquad (3)$$

2. *Trail and error method*

   The difference between the two tested crystals (LSO, LuYAP) mean (μ_{LuYAP} -μ_{LSO}) is calculated and divided into twenty steps of iteration. Starting from μ_{LSO} and ending at μ_{LuYAP} the correct identification percentage is calculated using equation (4) and reported at each threshold value as shown in fig 8. The desired threshold value is selected at least error (best correct percentage).

For the purpose of this investigation study; there are four modes of operation resulted form threshold calculation stage and can be summarized as the first two modes are using 50 pulses of each crystals to calculate the threshold by statistical equation and trail and error methods. The second two modes are using 500 pulses of each crystal to calculate the threshold by statistical equation and trail and error methods.

Third: The percentage of correct identification has been defined by

$$\%correct = \frac{1}{2} \times \left(\frac{N^{LSO}_{correct}}{N^{LSO}_{All}} + \frac{N^{LuYAP}_{correct}}{N^{LuYAP}_{All}} \right) \times 100\% \qquad (4)$$

Where the $N_{correct}$ and N_{All} represent respectively the numbers of correctly identified and all recorded pulses of material specified by the upper index.

Finally, a DWT coefficient is subtracted from the desired threshold and a greater-than-zero decision gives a discrimination answer of the two decay pulses. If the result is greater than zero then a slow pulse (LuYAP), otherwise a fast pulse (LSO) [2].

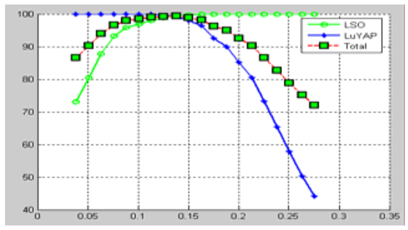


Fig.8 the fraction of correct identifications is shown for various values of threshold

The results of the proposed PSD technique for the 10K processed pulses (5000 pulses of each type (LSO and LuYAP) are shown in Fig. 9.

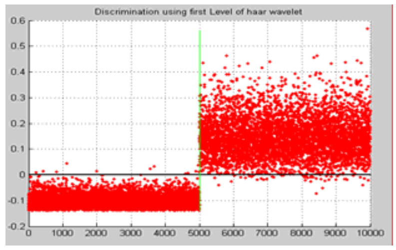


Fig.9 the DWT-based PSD technique applied to 5K pulses of each LSO and LuYAP pulses, respectively

Fig. 10 shows the summary of procedure flow chart. First, the pulse is low pass filtered by anti-aliasing filter, and converted to digital samples by analog to digital converter (ADC) at 40 MHz, and normalized. The Threshold is calculated in the training stage only. Second, the DWT of the digital pulse is calculated. Third, a threshold subtraction step is calculated to subtract DWT coefficient from its threshold relative value. Finally, a greater-than-zero decision, (Diff.>0?), gives a discrimination answer as described above.

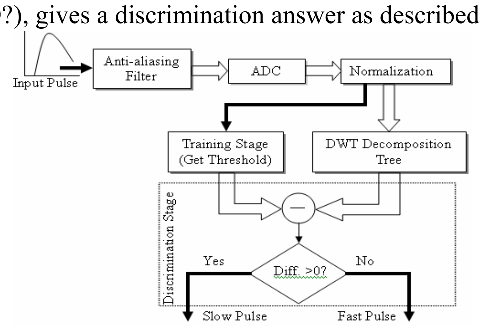


Fig.10 the flow chart of the investigated pulse shape discrimination technique

V. Experimental Results

Fig. 11 is a summary of all the different experimental conditions and shows the sample group which processed with 33 mother wavelet shown in table 2 under different modes of operation. For each DWT decomposition level, different parameters are tested. These parameters are threshold value, coefficient number at which best correct % occurred, decomposition level number at which best correct % occurred, best correct %, and execution time of calculating the DWT. The execution time results shown in table 2 were achieved on a Windows Laptop with a 2 GHz Intel core2due processor. It is dependant on the wavelets family and independent on the other parameters of experiment.

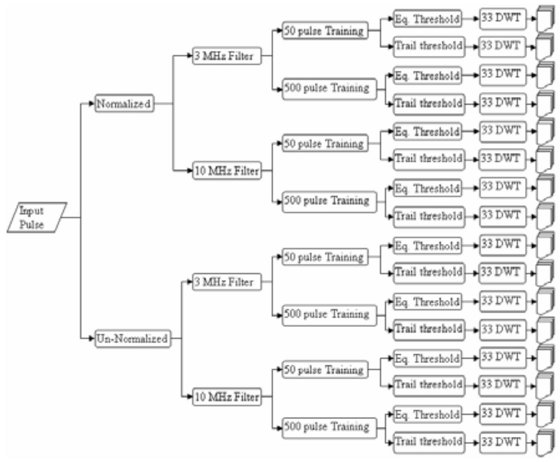


Fig.11 the flow chart of the experiment

The interested results are shown in tables 3 to 6. The results from using 1% of the data to calculate the threshold are worst than using 10% of the data. Hence, they are excluded. It can be explained as this small training data could not represent all pulses cases (these cases resulted from noise added to pulses, or to the current PET scanner problems such as parallax error, higher pixelization, and energy thresholding adjusted to crystal granularity).

Table 3 Results for 3 M filter Normalized data using 500 pulses with trail and error and Statistical equation methods to calculate threshold.

WVT Family	Trail and error method				Statistical Equation Method			
	Correct %	Level #	Threshold	Coeff. #	Correct %	Level #	Threshold	Coeff. #
bior2.6	99.75	3	-0.0916	8	99.66	3	-0.1243	8
sym8	99.74	3	0.2176	8	99.56	3	0.4260	12
coif1	99.71	3	0.2196	6	99.59	3	0.1861	6
sym5	99.71	2	0.2086	8	99.44	2	0.1829	8
db4	99.71	2	0.1888	8	99.48	3	0.3989	7
coif3	99.69	2	0.1959	13	99.58	3	0.2009	16
db7	99.69	2	0.1882	12	99.41	2	0.1632	12
coif5	99.66	2	0.14468	19	99.35	2	0.1010	19
bior2.8	99.66	3	0.3786	14	99.37	3	0.3183	14
bior1.5	99.65	3	0.0812	6	99.39	3	0.0138	6
sym3	99.65	3	0.1114	6	99.53	3	0.0486	6
db3	99.65	3	0.1114	6	99.53	3	0.0486	6
db6	99.63	1	0.1152	12	99.21	1	0.0959	12
db2	99.63	3	0.2614	4	99.19	1	0.1024	8
bior5.5	99.61	3	0.3635	8	99.31	2	0.1957	10
bior2.2	99.60	3	-0.0381	6	99.19	3	-0.1043	6
db8	99.60	3	0.0714	15	99.57	3	0.0051	15
db5	99.60	1	0.1161	11	99.14	1	0.0957	11
sym7	99.58	1	0.1372	13	99.40	3	0.3373	10
bior3.9	99.57	3	-0.1689	10	99.52	3	-0.1851	10
sym6	99.53	3	0.3234	7	99.41	3	0.3256	8
bior1.3	99.50	3	0.2612	4	99.52	3	0.2467	4
bior3.5	99.47	3	0.3708	8	99.09	2	0.1612	9
coif2	99.45	2	0.1649	12	99.61	3	0.1828	8
bior6.8	99.42	3	0.0726	10	99.57	3	0.3176	14
bior3.1	99.42	1	0.1081	8	98.60	2	0.1250	5
haar	99.41	1	0.1388	7	99.29	2	0.1433	4
bior3.7	99.40	1	0.1091	11	99.17	3	0.2228	9
coif4	99.39	3	0.1459	17	99.56	3	0.2084	21
bior4.4	99.39	3	0.1766	9	99.20	3	0.2537	8
bior3.3	99.38	1	0.1082	9	99.26	3	0.1884	6
dMeyr	99 36	3	-0.0085	30	99.59	3	-0.005	1
bior2.4	99.26	1	0.1288	9	99.17	2	0.1357	9

Table 4 Results for 10 M filter Normalized data using 500 pulses with trail and error method to calculate threshold (Res 10M Norm 500 trail)

WVT Family	Correct %	Level #	Threshold	Coef. #
bior2.4	99.72	3	0.0000204	6
dMeyr	99.69	3	0.1067965	47
bior1.3	99.69	3	0.0888492	4
bior3.9	99.68	3	-0.169431	10
coif3	99.65	3	0.106698	16
coif5	99.63	3	0.0911824	1
bior4.4	99.63	3	0.1588048	6
haar	99.63	3	0.2247442	2
sym6	99.61	3	0.2039803	6
bior6.8	99.59	3	0.1914297	14
coif4	99.57	3	0.1030695	15
coif1	99.49	3	0.1418436	6
bior5.5	99.47	3	0.2064642	10
bior2.8	99.45	3	0.0433606	16
db8	99.45	3	0.0398443	15
sym5	99.42	2	0.1287171	9
db7	99.42	2	0.1339968	12
bior2.6	99.41	3	0.1831731	11
db6	99.41	3	0.2412093	10
coif2	99.38	3	0.1017772	8
db4	99.38	2	0.1352942	8
bior3.7	99.25	3	0.2446249	10
bior1.5	99.21	2	0.1236449	8
db5	99.21	2	0.1417134	9
sym8	99.17	3	0.1257849	8
sym3	99.13	3	0.036442	6
db3	99.13	3	0.036442	6
sym7	99.03	3	0.0655021	9
db2	99.03	2	0.1248947	5
bior3.3	98.86	3	0.1451015	6
bior2.2	98.38	3	0.0827993	6
bior3.5	98.14	2	0.1205464	9
bior3.1	96.47	1	0.0787504	8

Table 5 Results for 3 M filter Un-normalized data using 500 pulses with trail and error method to calculate threshold (Res 3M UN Norm 500 trail)

WVT Family	Correct %	Level #	Threshold	Coef. #
bior3.9	99.81	3	-56.43883	10
bior2.6	99.80	3	-27.575478	8
dMeyr	99.79	3	-27.799619	59
bior2.2	99.72	3	-19.068217	6
db8	99.62	3	17.733272	15
bior6.8	99.48	3	15.859923	10
bior1.5	99.48	3	20.62428	6
bior3.3	99.42	3	-116.10741	5
sym3	99.38	3	17.341854	6
db4	99.38	3	-8.3321978	8
db3	99.38	3	17.341854	6
sym5	99.37	3	-1.9240092	6
bior2.4	99.36	3	-12.272646	9
sym7	99.29	3	4.7646004	9
bior2.8	99.26	3	-74.649855	10
coif5	99.17	3	30.786323	1
sym8	99.13	3	16.216505	10
db5	98.87	3	3.4359307	9
coif4	98.85	3	24.370224	17
bior4.4	98.58	3	32.162223	9
coif2	98.47	3	33.81266	11
bior3.1	98.46	3	-141.84376	3

sym6	98.46	3	39.299702	11
bior3.5	98.42	3	-99.816942	7
coif3	97.43	3	50.858762	12
db6	97.16	3	17.800014	11
bior3.7	96.35	2	-184.81041	2
coif1	95.89	3	65.733709	5
db2	95.39	2	49.880115	6
bior1.3	95.25	2	50.075595	6
db7	93.88	1	40.103547	13
haar	93.14	1	41.170939	8
bior5.5	92.98	1	41.504249	11

Table 6 Results for 10 M filter Un-normalized data using 500 pulses with trail and error method to calculate threshold (Res_10M_UN_Norm_500_trail)

WVT Family	Correct %	Level #	Threshold	Coef. #
bior2.4	99.66	3	-2.955346	6
dMeyr	99.51	3	-20.28772	1
bior3.9	99.51	3	-101.7908	10
coif4	99.48	3	28.593219	4
db8	99.31	3	22.999256	15
coif5	99.08	3	40.078451	1
bior2.6	99.07	3	-11.07973	8
sym3	99.02	3	20.465082	6
db3	99.02	3	20.465082	6
sym6	98.87	3	32.023857	11
db7	98.66	2	-66.73565	6
coif3	98.65	3	44.451348	16
bior1.3	98.50	3	53.96049	4
bior1.5	98.46	3	35.532812	6
coif2	98.26	3	53.403628	8
sym7	98.24	3	27.250586	9
bior6.8	97.88	3	48.582926	10
sym5	97.84	2	2.3579174	6
sym8	97.71	3	57.944695	14
bior3.7	97.60	3	-58.98573	11
bior2.8	97.42	3	32.14836	16
bior2.2	97.22	3	36.371708	6
bior3.3	97.16	3	-109.8574	5
db4	97.02	3	14.341928	8
coif1	96.75	3	68.086875	6
db5	94.01	3	20.242736	9
bior4.4	92.91	3	76.901268	9
bior3.5	92.73	2	-569.6196	2
bior3.1	86.78	3	-461.5673	1
db6	85.46	2	59.003527	11
db2	84.88	1	-25.52445	2
haar	78.29	1	44.596659	8
bior5.5	78.10	1	49.970975	11

VI. Discussion And Conclusion

Comparing the computing correctness results from both threshold computing methods, the trail and error method is better than the statistical equation for most the wavelet families see table 3. Thus, the statistical equation needs to be adapted to reach the better performance. This error comes from that the normal fitting of the LSO and LuYAP data distribution is not ideal normal $\sigma_{LSO} \neq \sigma_{LuYAP}$. The normal distribution of LSO data is sharper than normal distribution of LuYAP data (LSO is faster than LuYAP). Thus the equation must be modified to move the threshold closer to LSO distribution, which will be investigated in the future work.

Using the normalized data, the correctness results are more stable for most the wavelet families and independent of tolerance of Nyquist filter. This is shown in tables 5, 6.

Notice tables 2, 3, 4, it is clearly noticed that 10 MHz Nyquist filter gives similar results to 3 MHz Nyquist filter but occurs at higher wavelet decomposition level and requiring higher execution time.

The correctness performance rang is from 99.75% to 99.26% for 3 MHz filtered data. On the other hand, for 10 MHz filtered data this range is from 99.72 to 96.47.

For the best performance , the 3 MHz filtered normalized data, and trail and error threshold using 10% of the data are considered.

Again notice table 3, the best result mostly occurred at the last coefficient which corresponds to the last sample in the pulse that indicate the decay of the pulse.

The highest correctness occurs with biorthogonal wavelet family, and its results varying from 99.75% (bior2.6) to 99.26% (bior2.4). 10 types of this family reach level 3 of wavelet decomposing to give its best results.

The second family is symlets, which ranges from 99.74% (sym8) to 99.53% (sym6). Three types of this family reach level 3 of wavelet decomposing to give its best results.

The third is coiflets and daubechies. The coiflet ranges from 99.71% (coif1) to 99.39% (coif4). On the other hand, daubechies ranges from 99.71% (db4) to 99.6% (db5).

The Biorthognal and symlets wavelet families are more complex in computation than the other families and need higher execution time.

The Daubechies algorithm has an overlap between iterations in the Daubechies transform step. This overlap allows the Daubechies to pick up detail. Coiflets wavelets are compact and oscillatory, so it is suitable for de-noising operation. Although, Coiflets and daubechies results are approximately similar but Coiflets need higher total execution time than daubechies.

Finally, The best compromise between performance and Discrete wavelet transform (DWT) decomposition level and execution time turned out that Dubechies 6 (db6) gives 99.63% successful discrimination rate at level 1 and consuming 12.828 sec over 10 000 pulses. Thus DB6 is recommended by this investigation to be the best wavelet family for discrimination.

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