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Citation: AIP Conference Proceedings **455**, 797 (1998); doi: 10.1063/1.57283
View online: http://dx.doi.org/10.1063/1.57283
View Table of Contents: http://aip.scitation.org/toc/apc/455/1
Published by the American Institute of Physics

Statistical deliberations for exotic nuclei

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Abstract. An often encountered situation in experiments on nuclei far from stability is that the count number per channel in various differential spectra becomes small. This might require the use of not so well-known statistical methods. Some examples are given.

INTRODUCTION

The question of what statistical method to use in the analysis of experimental data has been considered in many textbooks, e.g. [1,2]. Experiments with nuclei far from stability nevertheless often result in situations where standard statistical methods do not immediately apply, partly due to the low count numbers that often are encountered. The aim of the present contribution is through three different illustrative examples to show how one then can proceed. The topics treated are biases in fit methods for Poisson distributed data, how to employ simple estimators in line shape analysis and efficient estimation of half-lifes.

LOW COUNT NUMBERS

When low count numbers occur one must use the Possion distribution explicitly rather than doing a Gaussian approximation. Since the Possion distribution is of exponential type the maximum likelihood will be an optimal method of analysis [2]. The often employed least squares minimization (χ^2-fit) gives a biased result. In two different variations of this method the count numbers or the theoretical values are used to estimate the error on the data points so that $\chi^2 = \sum_i (n_i - \theta_i)^2/n_i$ or $\chi^2 = \sum_i (n_i - \theta_i)^2/\theta_i$. Both methods gives a bias that asymptotically will be -1 and $1/2$, respectively [3]. For "experimental error bars" this follows since points below the "true value" will be attributed a too small error, conversely for points above. For "theoretical error bars" this follows since the denominator in χ^2 on its own would force θ_i to become large. A detailed derivation of the bias can be found in [3]. For a large number of channels N_0 with common "true value" μ the two variants of χ^2-minimization gives fit-parameters that are approximately $\mu^2/(\mu+1)$

CP455, *ENAM98: Exotic Nuclei and Atomic Masses*
edited by B. M. Sherrill, D. J. Morrissey, and Cary N. Davids
 1-56396-804-5/98/$15.00

and $\mu\sqrt{1+1/\mu}$, respectively. Results are also given in the limit of small N_0 and have in all cases been verified by means of Monte Carlo methods. Note that a maximum likelihood minimization not only gives the correct result for all N_0, but also gives the smallest variance of the estimate. It should therefore be used instead of least squares minimization whenever effects of order $1/n_i$ are judged important.

SIMPLE ESTIMATORS

Other methods than maximum likelihood and least squares are occasionally preferable, e.g. when the exact underlying distribution is not well established and one at first can be satisfied with obtaining only a few characteristics of the data. This situation might be encountered when the total statistics is limited and/or does not allow a detailed line shape analysis to be made, but one nevertheless would like to extract some main quantities. A way out is to resort to simple (distribution-free) methods [1,2]. As a practical example we can refer to data from a recent experiment at GANIL [4] where the aim is to obtain the width for the neutron longitudinal momentum distribution following break-up of ^{10}Be and ^{11}Be in a Be target.

Let the total number of counts be N. All summations should be understood to run over all counts, i.e. $\sum_i f_i$ should be interpreted as $\sum_{i=1}^{N} f_i$ or if the data are binned as $\sum_{i=\text{all bins}} n_i f_i$, where n_i is the number of counts in bin i ($\sum_i n_i = N$). For each count a "position variable" is determined as x_i, for binned data this is taken as the position in the middle of the bin. The data points are assumed to stem from some distribution that has mean value μ and variance V.

The location parameter can be estimated by $\hat{\mu} = \bar{x}$, the average value of x_i. The similar unbiased estimate of the variance is well-known to be:

$$\hat{V} = \frac{N}{N-1}\overline{(x-\bar{x})^2} = \frac{1}{N-1}\sum_i (x_i - \bar{x})^2 \ ,$$

and an error estimate for $\hat{\mu}$ is then given by $V(\hat{\mu}) = \hat{V}/N$. The corresponding procedure for obtaining an error estimate for the width [1] is less well known. A relatively simple derivation gives the following unbiased estimators:

$$\mathrm{cov}(\hat{\mu}, \hat{V}) = \frac{1}{N-2}\frac{1}{N-1}\sum_i (x_i - \bar{x})^3 \ ,$$

$$V(\hat{V}) = \frac{N}{N^2-3N+3}\frac{1}{N-1}\sum_i (x_i - \bar{x})^4 - \frac{1}{N}\left(3\frac{2N-3}{N^2-3N+3} + \frac{N-3}{N-1}\right)\left(\frac{1}{N-1}\sum_i (x_i - \bar{x})^2\right)^2 \ ,$$

where also the covariance between $\hat{\mu}$ and $\hat{V}$ is included. The statistical efficiency of these simple estimators will of course depend on what the exact underlying

distribution is (it might even asymptotically go to zero, cf. the discussion in section 8.7 in [2]), so if independent reliable knowledge on the distribution is available one should incorporate it by using more elaborate methods.

For the case in question the above formulas can be applied directly to extract parameters for the distribution from ^{10}Be (core) break-up. The position decreases and the width increases significantly as one goes to larger angles, i.e. larger transverse momenta. For break-up of ^{11}Be one might assume a two-component distribution: a contribution from the core (the one just extracted) and a contribution from the halo neutron. Using the ^{10}Be data a disentanglement can be done both for the position and for the width variables, but only the latter could be extracted with sufficient accuracy for the halo component. It is clearly smaller than the core width (the two variances differ by about a factor two) and also shows clear indications for an increase with angle. To draw more detailed physical conclusions one needs to introduce models for the process at this point.

HALF-LIFE DETERMINATIONS

Finally, the case of half-life determinations will be considered in some detail. The maximum likelihood is here again an optimal method of analysis also in the limit of low count numbers and can be used for finding parameters and their errors. As is well-known [1,2] the best estimator for the mean life τ is the average time $\bar{t}$ (multiply with $\ln 2$ to get the half life instead). If data only are taken up to an upper limit T one must iterate the equation

$$\tau = \bar{t} + \frac{T}{e^{T/\tau} - 1}$$

to obtain the mean life. If furthermore the data are binned with a bin width of Δt the equation to be iterated turns into

$$\tau = \bar{t} + \frac{T}{e^{T/\tau} - 1} + \left(\tau - \frac{\Delta t}{e^{\Delta t/\tau} - 1}\right) .$$

The variance is similarly given by explicit formulas. It is τ^2/N (where N is the number of data points) in the simplest case and increases for finite T and for increasing Δt.

There are many powerful statistical methods for determining the fit-quality for an underlying exponential distribution [5]. As an example the Anderson-Darling statistics that is based on the empirical distribution function will be employed here to illustrate how the half life can be determined for ^{31}Ar from the observed time distribution of beta-delayed two-proton events [6]. The activity was produced at ISOLDE by a pulsed proton beam and diffuses gradually out of the target. The ions are collected while the decays are recorded and the goodness-of-fit test is therefore needed to ensure that one has reached a regime of "pure exponential decay" before a reliable value can be extracted. As illustrated in figure 1 this happens about 85 ms

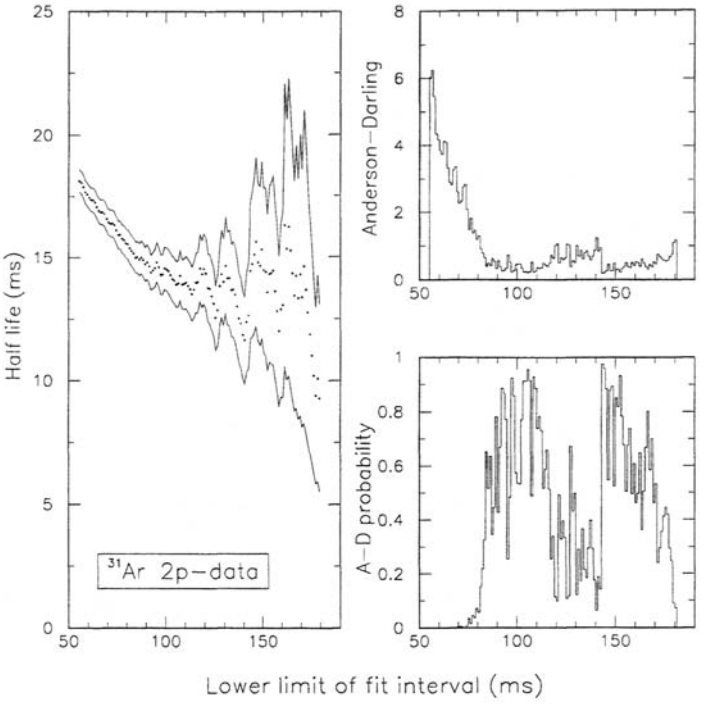

FIGURE 1. Half life of ^{31}Ar determined from two-proton data for different fit-intervals as function of the starting point of the interval. Half life is found with maximum likelihood and goodness-of-fit is done with the Anderson-Darling statistics.

after start of collection (the "A-D probability" measures the agreement between the data and a single exponential distribution). Only after this point can the extracted half life values be trusted.

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