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Some Remarks on the Error Analysis in the Case of Poor Statistics

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A prescription for the error analysis of experimental data in the case of stochastic background is formulated. Several relations are given which allow to establish the significance of mother-daughter relationships obtained from delayed coincidences. Both, the probability that a cascade is produced randomly and the probability that the parameters of an observed event chain are incompatible with known properties of a given species are formulated. The expressions given are applicable also in cases of poor statistics down even to single events.

1. Introduction

Particularly in nuclear physics, the technique of delayed coincidences is widely used for detecting time correlations between signals of different groups. Previously unknown isomeric states as well as new nuclides may be identified by linking them to the subsequent decays of their known daughter nuclei. In many cases, α decay cascades were used for this purpose [1–10].

There are two aspects in establishing the significance for the existence of a true correlation:

- Consideration of the possibility that the random background of uncorrelated events could simulate a correlation.
- Estimation of the compatibility of the parameters of observed events with known properties of some members in the considered event chain. In the following, both aspects are discussed.

Many of the relations given here are not entirely new, the intention, however, is to compile the probability arguments relevant for the correlation analysis so as to be easily accessible. Finally examples for the application in recent experiments are given as an illustration.

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2. The Definition of the Error Probability in the Case of Stochastic Background

In most cases it is not possible to give the probability for a statement to be true because then the complete set of error possibilities would have to be known. But a special error possibility of the statement can always be defined and the probability for the occurrence of this error can be estimated.

In experimental nuclear physics, often some sort of stochastic background is present which may be a possible source of errors. This background may easily be treated mathematically by use of Poisson's distribution.

Let us consider a measurement in which the value x of a physical quantity X is determined for single events. The measured spectrum dn/dx is schematically shown in Fig. 1. Part of the spectrum is similar to the expected frequency distribution $f(x)$ of background events which is assumed to be known. A group of n_m events with $x \geqq x_{\min}$, however, may have a different (perhaps more interesting) origin. $x_{\min}$ is the value of the variable x of the events to be investigated which is nearest to the mean value of the background. (In the example of Fig. 1, the interesting events are located in the right tail of the background distribution.) The probability that the n_m events are produced by a stochastic fluctuation of

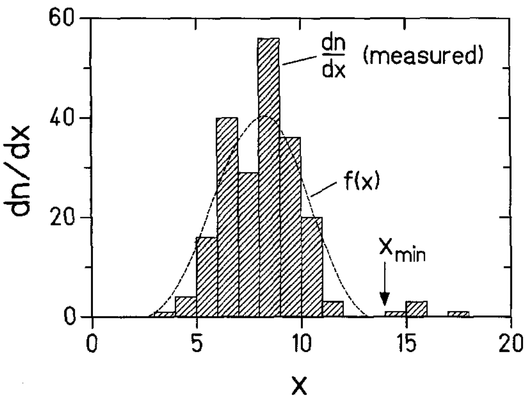

Fig. 1. Schematic representation of a measured spectrum of the variable x. The expected frequency distribution $f(x)$ of background events is indicated. The probability that the group of events with $x \geqq x_{\min}$ can be understood as a stochastic fluctuation of the background distribution gives the error probability of a different (more interesting) interpretation of these events

the background distribution is identical to the error probability of a different interpretation of these events, if only this special error possibility is considered.

The expectation value n_b for the number of events, deviating at least as much from the mean value of the statistical frequency distribution as any of the events to be investigated, is given by integrating the background frequency distribution:

$$n_b(x \geqq x_{\min}) = \int_{x_{\min}}^{\infty} f(x)\,dx \tag{1}$$

The probability that a fluctuation of the background distribution produces the observed number of events n_m or more in the interval from $x_{\min}$ to infinity is given by the sum over Poisson's distribution:

$$P_{\mathrm{err}} = \sum_{n=n_m}^{\infty} \frac{n_b^n}{n!} e^{-n_b}. \tag{2}$$

This is the error probability searched for.

If the condition $n_b \ll 1$ is fulfilled, the following approximation can be made:

$$P_{\mathrm{err}} \approx \frac{n_b^{n_m}}{n_m!}. \tag{3}$$

3. Correlation Analysis

In the case of a correlation analysis, the time distribution of the events has to be investigated. Other observed physical quantities of the events (e.g. pulse height, pulse shape) may be used to identify the members of different event groups. An event group may be defined by some decay mode of a specified kind of nuclei. The occurrence of events of the different groups form independent Poisson processes.

A true correlation between members of different event groups is characterized by the occurrence of time distances between the members of a correlation chain shorter than expected for the random distribution. Therefore we may choose an arbitrary limit Δt which divides the measured spectrum of time distances in an upper random group and a lower group of n_m events which may contain correlated events. A correlation can be established if there is only a small probability to produce the observed number of n_m events with time distances $t \leqq \Delta t$ as a stochastic fluctuation of the random distribution. The expressions for some special cases are formulated in the following.

3.1. Correlation Chain with Fixed Order

Suppose that in an experiment at least K different event groups are defined. We assume that n_m event chains, consisting of one event E_1 of the first group followed by one event E_2 of the second group and so on, and finally by one event E_K of the K'th group, were observed. The probability density that stochastically an event E_i is followed by an event E_{i+1} after the time distance t, is given by the random time interval distribution of events E_{i+1} times the probability of not observing an event of any other group in between:

$$dp_{i,i+1}/dt = \lambda_{i+1} e^{-\lambda_{i+1} t} \prod_{j \neq i+1} e^{-\lambda_j t} = \lambda_{i+1} e^{-\sum_{j=1}^{K} \lambda_j t} \tag{4}$$

where λ_i is the mean counting rate of the event group E_i.

The probability to observe the sequence E_i, E_{i+1} within a time interval $\Delta t_{i,i+1}$ is given by integrating (4) from zero to $\Delta t_{i,i+1}$. As the events of the different groups are independent of each other, the expectation value for the number of the complete sequences as defined above within the time T is given by the product of all the probabilities for the sequences E_i, E_{i+1} (with $i=1$ to $K-1$) times the number n_1 of events of type 1 ($n_1 = \lambda_1 T$):

$$n_b = \lambda_1 T \prod_{i=1}^{K-1} \left\{ \int_0^{\Delta t_{i,i+1}} dp_{i,i+1}/dt\, dt \right\}$$

$$= T \frac{\prod_{i=1}^{K} \lambda_i}{\left(\sum_{i=1}^{K} \lambda_i\right)^{K-1}} \prod_{j=1}^{K-1} \left\{1 - e^{-\sum_{i=1}^{K} \lambda_i \Delta t_{j,j+1}}\right\} \tag{5}$$

$\Delta t_{i,i+1}$ are maximum time limits given for the sequences E_i, E_{i+1}.

If the conditions $\sum_{i=1}^{K} \lambda_i \Delta t_{j,j+1} \ll 1$ are fulfilled for all possible j values, n_b can be approximated:

$$n_b \approx T \prod_{i=1}^{K} \lambda_i \prod_{i=1}^{K-1} \Delta t_{i,i+1} \tag{6}$$

The error probability is calculated as formulated in Sect. 2.

3.2. Correlation Chain with Partially Free Order

The a priori knowledge of the order of the events in a possibly true event chain may be limited. There are many different possibilities for an incomplete knowledge. Therefore we only formulate one case which is characterized by the condition that possible decay sequences are known to start with the events E_1 of the group 1. The events of the other event groups (E_2 to E_K) may appear in any order, but at least one event E_i must appear within the time limit $\Delta t_{1,i}$. In a consideration similar to Subsect. 3.1 we obtain the expectation value for the number of complete sequences to be produced randomly:

$$n_b = \lambda_1 T \prod_{i=1}^{K-1} \left\{ \int_0^{\Delta t_{1,i+1}} dp_{1,i+1}/dt\, dt \right\}$$

$$= T \frac{\prod_{i=1}^{K} \lambda_i}{\prod_{j=2}^{K} (\lambda_1 + \lambda_j)} \prod_{i=2}^{K} \{1 - e^{-(\lambda_1 + \lambda_i)\Delta t_{1,i}}\} \tag{7}$$

If the conditions $(\lambda_1 + \lambda_i)\Delta t_{1,i} \ll 1$ are fulfilled for all possible j values, the following approximation can be made:

$$n_b \approx T \prod_{i=1}^{K} \lambda_i \prod_{i=2}^{K} \Delta t_{1,i}. \tag{8}$$

4. The Concept of the Central Confidence Interval

In a second step the identification of an established decay sequence is treated by comparing its properties with known properties of previously investigated members of the sequence. For example the physical quantity X for which the value x_m was measured is known by spectroscopic investigations to have the expectation value x_0 if the identification is correct. The compatibility between x_m and x_0 is investigated.

Since we want to treat cases of poor statistics, we cannot apply the χ^2 test. Therefore we use the concept of central confidence intervals. For a confidence level $1-\varepsilon$, this means that the error probability (the probability that the true value is not included in the confidence interval around x_m) is smaller than or equal to $\varepsilon/2$ on each side outside this interval. We restrict ourselves to one-dimensional distributions which decrease monotonically with increasing distance from the most probable value and which depend on one parameter. If $p(x|\mu)$ is the probability density of the random variable x with the parameter μ, the central confidence limits μ_1 (lower limit) and μ_u (upper limit) are given by the solutions of the following equations [11]:

$$\sum_{x=x_m}^{\infty} p(x|\mu_l) = \varepsilon/2$$
$$\sum_{x=-\infty}^{x_m} p(x|\mu_u) = \varepsilon/2 \tag{9}$$

x_m is the actually measured value of x. In the case of a continuous variable x, the summation is replaced by an integration.

The values μ_l and μ_u determine a confidence interval for the expectation value of the random variable x. (For many distributions $p(x|\mu)$ of practical interest the values of the parameter μ and of the expectation value of x are identical.)

The identification of a correlation chain can only be accepted, if the confidence interval with a certain confidence level $1-\varepsilon$ (e.g. $1-\varepsilon=0.68$) around the measured value x_m of the variable x to be tested includes the known value x_0 of the suspected species.

5. The Compatibility of Two Experimental Values

In the more general case, the presently measured value x_{m1} of the physical quantity X has to be compared to a previously measured value x_{m2}. We want to test the hypothesis, that both measured values belong to the same family of distributions $p(x|\mu, v_i)$ with the same parameter μ and possibly different parameters v_i. (For example two different measured values for an α decay energy may have different uncertainties. In this case two gaussians with different width parameters v_i and the same mean value μ may be taken.) This hypothesis can be accepted if there is a great chance that the variable x_1 of the distribution $p(x|\mu, v_1)$ is smaller than x_{m1} and that simultaneously the variable x_2 of the distribution $p(x|\mu, v_2)$ is greater than x_{m2}. A measure for the rejection probability is given by minimizing the following expression with respect to μ:

$$P_{rej}=1-\frac{\int_{-\infty}^{x_{m1}} p(x|\mu,v_1)\,dx \int_{x_{m2}}^{\infty} p(x|\mu,v_2)\,dx}{\int_{-\infty}^{x_0} p(x|\mu,v_1)\,dx \int_{x_0}^{\infty} p(x|\mu,v_2)\,dx} \tag{10}$$

It is assumed that the expectation value of the distribution $p(x|\mu,v)$ does not depend on the value of v. The integrals in the denominator take into account the implicit restriction that x_{m1} and x_{m2} are allowed to deviate each only to one side of the expectation value x_0.

$$x_0=\int_{-\infty}^{\infty} x\,p(x|\mu,v_1)\,dx=\int_{-\infty}^{\infty} x\,p(x|\mu,v_2)\,dx$$

For μ that value has to be taken, which minimizes P_{rej} as given by (10). It was assumed that $x_{m1}\leqq x_{m2}$. In the opposite case, the integration limits in (10) have to be interchanged.

For symmetric distributions the result can be formulated by the error probabilities ε_1 and ε_2 of central confidence intervals belonging to the two distributions with the same parameter μ which just reach to the appropriate measured value x_{m1} and x_{m2}, respectively. In this case P_{rej} is given by minimizing the following expression with respect to μ:

$$P_{rej}=1-\varepsilon_1\,\varepsilon_2$$

The present definition of P_{rej} differs from the treatment often found in literature which considers the distribution of differences $|x_2-x_1|$ between one random variable x_1 of the distribution $p(x|\mu,v_1)$ and one random variable x_2 of the second distribution $p(x|\mu,v_2)$. In the present definition the absolute values determined in the measurement are used which are disregarded if the distribution of differences is considered.

In any correlation analysis two measured quantities occur: the number of the possibly correlated events and their mean time distance. The first one must agree with the knowledge of the decay branches and the detection probability for decay chains and the second must be compatible with the knowledge of the life time of the consecutive event group. Therefore both will be discussed in the following.

6. The Number of Decay Sequences

For statistically independent events, the probability to observe n events if the mean value of the number of events is μ, is described by the Poisson distribution:

$$p(n|\mu)=\frac{\mu^n}{n!}e^{-\mu} \tag{11}$$

The confidence limits for an observed number of n_m counts are implicitly given by the following equations:

$$\begin{aligned}\sum_{n=n_m}^{\infty} p(n|\mu_l)&=1-\sum_{n=0}^{n_m-1}\frac{\mu_l^n}{n!}e^{-\mu_l}=\varepsilon/2\\ \sum_{n=0}^{n_m} p(n|\mu_u)&=\sum_{n=0}^{n_m}\frac{\mu_u^n}{n!}e^{-\mu_u}=\varepsilon/2\end{aligned} \tag{12}$$

For $n_m\geqq 2$, a good approximation for the confidence limits is given by the following relations:

$$\mu_u\approx n_m+z(1+\sqrt{n_m}),\qquad \mu_l\approx n_m-z\sqrt{n_m} \tag{13}$$

The quantity z is related to the chosen confidence level $(1-\varepsilon)$ by the following integral over the density of the normal distribution:

$$\frac{\varepsilon}{2}=\int_z^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx \tag{14}$$

This is a convenient prescription for estimating errors of variables which are described by Poisson statistics, e.g. cross sections, if only a small number of events was observed. It even gives a reasonable estimate for the upper limit in the case of one observed event.

For small numbers, this approximation for the upper limit is much more accurate than the generally used symmetric error (see Fig. 2):

$$\mu_u-n_m=n_m-\mu_l=z\sqrt{n_m}$$

The error limits in the cases which cannot be described by the approximation (13) have to be calculated exactly. The standard errors $(z=1)$ for $n_m\leqq 2$ are given in Table 1.

Table 1. Standard errors for small numbers. Standard errors (confidence level $1-\varepsilon=0.68$) of the expectation value of the Poisson distribution and the exponential distribution for small numbers n of observed events, calculated with relations (12) and (17). For $n=0$ the upper limit μ_u corresponds to an upper error probability of $\varepsilon=0.16$. In cases where the approximations (13) and (18) are accurate within 10%, the exact values are given in parentheses

Number of counts	Poisson's distr.		Exponential distr.	
n	μ_l	μ_u	τ_l/t_m	τ_u/t_m
0	0	1.84	–	–
1	0.173	(3.30)	(0.543)	5.79
2	(0.708)	(4.64)	(0.606)	(2.82)

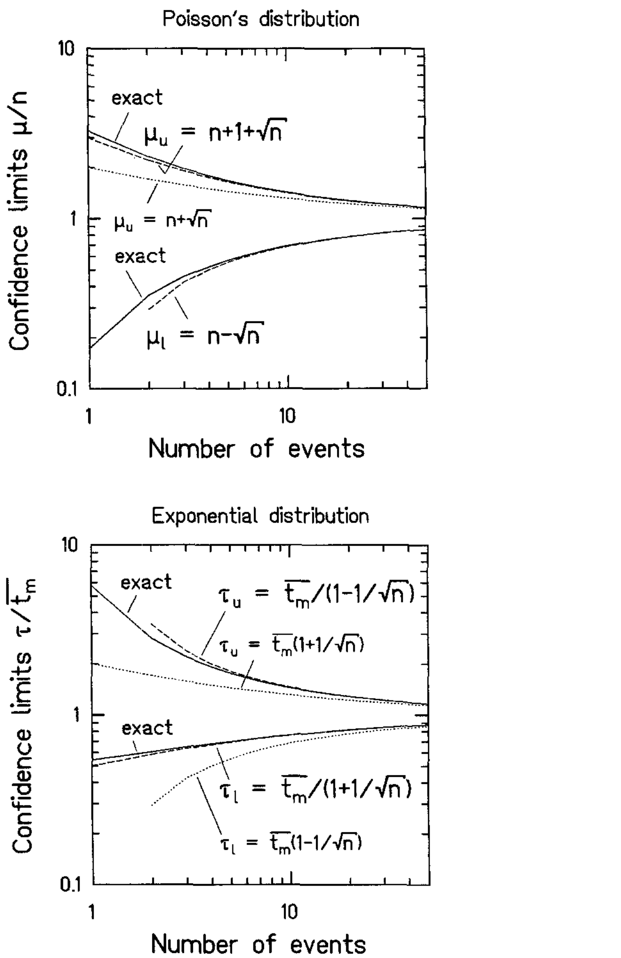

Fig. 2a and b. Central confidence intervals for a confidence level of 68% (which corresponds to the standard deviation of a gaussian) for two different distributions as a function of the number n_m of observed events. Figure 2a applies to a variable following Poisson's distribution and Fig. 2b applies to a variable following an exponential distribution. In addition to the exact curves, two kinds of approximations are shown: The dotted curves show the conventional approximations based on the idea that the standard deviation of n events converges to $\sqrt{n}$ for large n. The much more accurate approximations are shown by the dashed curves. The formulae are given in the figure. The few exceptions which cannot be described by the approximative formulae are given in Table 1

7. The Life Time of the Daughter Species

The maximum likelihood estimate of the life time τ is the arithmetic mean $\bar{t}_m$ of the individual life times $(t_m)_i$ at which events were observed (if the observation time is not restricted):

$$\bar{t}_m = 1/n \sum_{i=1}^{n} (t_m)_i \tag{15}$$

For the calculation of confidence limits, the distribution of the quantity $\bar{t} = 1/n \,\Sigma\, t_i$ is needed. For a given life time τ, the probability density of the random variable $\bar{t}$ is given by the product of the probability densities of the individual time values t_i, integrated over all combinations which conserve the mean value $\bar{t}$. Making use of the δ function the probability density can be written [14]:

$$p_n(\bar{t}|\tau) = \int_0^\infty \dots \int_0^\infty \prod_{i=1}^{n} \left[\frac{1}{\tau} e^{-t_i/\tau}\right] \delta\left(\bar{t} - \frac{1}{n}\sum_{i=1}^{n} t_i\right) dt_1 \dots dt_n$$
$$= \frac{n^{n+1}}{n!} \frac{\bar{t}^{n-1}}{\tau^n} e^{-n\bar{t}/\tau} \tag{16}$$

The confidence limits τ_1 and τ_u are given by the solutions of the following equations if $\bar{t}_m$ is the actually observed value for $\bar{t}$:

$$\int_{\bar{t}_m}^{\infty} p_n(\bar{t}|\tau_l)\, d\bar{t} = \sum_{n=0}^{n-1} \left(\frac{n\,\bar{t}_m}{\tau_l}\right)^n \frac{1}{n!} e^{-(n\bar{t}_m)/\tau_l} = \varepsilon/2$$
$$\int_0^{\bar{t}_m} p_n(\bar{t}|\tau_u)\, d\bar{t} = 1 - \sum_{n=0}^{n-1} \left(\frac{n\,\bar{t}_m}{\tau_u}\right)^n \frac{1}{n!} e^{-(n\bar{t}_m)/\tau_u} = \varepsilon/2 \tag{17}$$

(The sum is obtained by partial integration.)
For $n \geqq 2$, the following approximations hold for the confidence limits [12]:

$$\tau_u \approx \frac{\bar{t}_m}{1 - z/\sqrt{n}}, \qquad \tau_l \approx \frac{\bar{t}_m}{1 + z/\sqrt{n}} \tag{18}$$

The quantity z is determined by the chosen confidence level $(1-\varepsilon)$ with relation (14). This is a convenient prescription for estimating errors of the mean life time if only a small number of events was observed. It even gives a reasonable estimate for the lower limit in the case of one observed event.
For small numbers, this approximation is much more accurate than the generally used symmetric error (see Fig. 2):

$$\bar{t}_m - \tau_l = \tau_u - \bar{t}_m = z\,\bar{t}_m/\sqrt{n}$$

The error limits in the cases which cannot be described by the approximation (18) have to be calculated exactly. The standard errors $(z=1)$ for $n_m \leqq 2$ are given in Table 1.

8. Two Illustrative Examples

The application of the correlation analysis as described above can best be illustrated by an exemplary treatment of experimental data. First we choose a case where the experimental technique [2] has already been published. We emphasize the description of the method for evaluating the data.

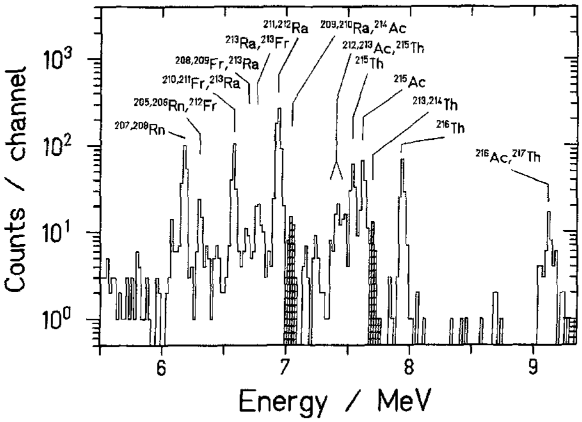

Fig. 3. Measured α spectrum of evaporation residues obtained in the reaction $^{96}Zr + ^{124}Sn$ (570 MeV)

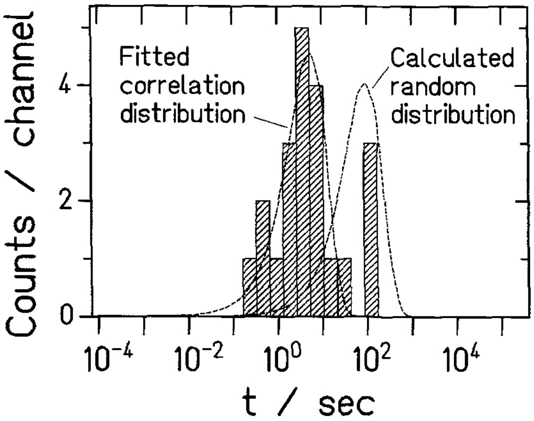

Fig. 4. Correlation analysis for the identification of ^{214}Th by the subsequent decays of the daughter nucleus ^{210}Ra. Histogram: spectrum of time distances between the events of α decays of mother and daughter nuclei. Dotted lines: fitted time spectrum for correlated decays together with a calculated time spectrum expected for purely random time distances. The probability to obtain the observed spectrum as a stochastic fluctuation of the random distribution is less than 10^{-3}

For our purpose, only the knowledge of a small part of the experiment is essential: ^{124}Sn projectiles with an energy of 570 MeV impinge on a target of ^{96}Zr. Evaporation residues are separated from the primary beam and directed to the detector position by the velocity filter SHIP [13]. Different kinds of unstable nuclei are implanted near the surface of a silicon surface barrier detector. Most of them decay by α radioactivity. The measured α spectrum is shown in Fig. 3. Among others two kinds of nuclei (^{214}Th and ^{210}Ra) are produced which differ by 2 neutrons and 2 protons. Both have a strong α decay branch. In this case, part of the observed α decays of ^{210}Ra which can be identified by their known transition energy may be the decay of daughter nuclei produced by the decay of ^{214}Th. In this way, a previous identification [15] of ^{214}Th which was made with the help of systematic arguments can be confirmed by a correlation analysis. In the measurement, the abolute time of each event is registered. Therefore for each event chain, the time distance between the decay of ^{214}Th and the consecutive decay of ^{210}Ra is known.

In the example, time distances between 20 ms and 15 s were registered. In many cases it is difficult to treat this time span with sufficient accuracy in the usual way by considering a linear time scale. One would need several thousands of time channels in order not to loose too much information if the time distances are sorted into a spectrum. Therefore we choose a spectrum with a logarithmic time scale. In this representation, the ideal radioactive decay curve becomes a peak with a universal shape independent of the life time. The frequency distribution of decay times

$$\frac{dn}{dt} = n\lambda e^{-\lambda t} \tag{19}$$

is changed to the following form, whereby the substitution $\ln(t) = \vartheta$ is used:

$$\frac{dn}{d\vartheta} = n\lambda e^{\vartheta} e^{-\lambda e^{\vartheta}} \tag{20}$$

Two free parameters – the number n of counts and the life time τ – determine the height and the position of this peak, respectively. The most probable value of this distribution is $\ln(\tau)$. In this way, the relevant information of each time distance in our example is included in 10 channels. This is demonstrated in Fig. 4.

In the spectrum of time distances, the expected random distribution (if no true correlation is present) is represented by a peak with the position $1/(\lambda_1 + \lambda_2)$ (see Eq. (4)) and the number of events $n \simeq n_1 n_2/(n_1 + n_2)$ (see Eq. (5)). In Fig. 4 the main intensity appears at shorter time distances whereas the random peak is much smaller than expected. With the relations given above, the probability for the left peak to be produced as a stochastic fluctuation of the random distribution can be estimated, if the numbers determined in the experiment are used: $n_1 = 28$, $n_2 = 38$, $T = 6080$ s. The limit Δt_{12} can be chosen arbitrarily. When the whole left peak with 18 events is included ($\Delta t_{12} = 40$ s), the error probability as given by relations (2) and (6) is $P_{err} < 10^{-3}$. In the range below $\Delta t_{12} = 5$ s, 12 events occur. The error probability for this part of the spectrum amounts to $P_{err} < 10^{-9}$.

The number n_m of observed events in this peak can be compared with the number n_c of expected correlated events:

$$n_c = q \times b_\alpha \times n_1 = 0.8 \times 1.0 \times 28 = 22.4$$

where q = apparative detection probability for daughter decays and $b_\alpha = \alpha$ branching of the daughter nucleus,

$$n_m = 18\,{}^{+5}_{-4}.$$

Within the confidence interval with the level $1-\varepsilon = 0.68$, both values are in agreement.

The mean time value t_c of the left peak in the measured spectrum of time distances has to be compared with the known half life $t_{1/2}$ of the daughter nucleus:

$$\tau = t_{1/2}/\ln 2 = 3.7\ \mathrm{s}/\ln 2 = 5.34\ \mathrm{s}$$

$$t_c = 5.4\,{}^{+1.7}_{-1.0}\ \mathrm{s}.$$

Again within the confidence interval with the level $1-\varepsilon = 0.68$, both values are in agreement.

Moreover, the measured mean α energy of daughter decays was found to be compatible with the known value for the decay of ^{210}Ra.

With the procedure shown in this example, the earlier [15], still somewhat tentative, identification of ^{214}Th can be confirmed. According to the spectroscopic information obtained in this experiment, ^{214}Th decays by α radioactivity with an α energy of (7,670 ± 20) keV and a half life of (96 ± 30) ms. These results agree with the values obtained in [4].

The second example we want to present deals with a single correlation chain involving several events. Recently, the observation of such a single characteristic decay chain was taken as an evidence for the first synthesis of an isotope of element 109 [16]. It was identified by determining its mass (with a limited resolution) and by observing the subsequent decay cascade of two alpha decays and one spontaneous fission. The identification relies on two aspects. First, one has to investigate the possibility to interpret the observed correlated event chain consisting of four events as a stochastic fluctuation of the time distances between background events. In a second step, the observed decay chain has to be assigned to a certain group of nuclei by comparing its characteristics with known spectroscopic properties.

In the first step, the error probability for a correlation chain with partially free order has to be evaluated. The impinging evaporation residue must be the first event whereas the fission decay has to be the last event in the cascade. The error probability may be estimated by using formulae (7) and (2). (As in formula (7) only the evaporation residue is fixed as first event, we will get an upper limit for the error probability.) The characteristics of the event chain as described in [16] allow to use the approximative formulae (8) and (3). Thus, the error probability is given by the following expression:

$$P_{\mathrm{err}} \approx T\lambda_1 \lambda_2 \lambda_3 \lambda_4 \Delta t_{1,2} \Delta t_{1,3} \Delta t_{1,4}. \tag{21}$$

By inserting the values for the counting rates λ_i of the different event groups, the total measuring time T and the observed time differences $\Delta t_{1,i}$ as given in [16], the following value is obtained:

$$P_{\mathrm{err}} \leqq 50 \times 250\ \mathrm{h} \times \frac{288}{50 \times 250\ \mathrm{h}} \frac{1}{50 \times 8\ \mathrm{h}} \frac{1}{50 \times 75\ \mathrm{s}} \frac{1}{50 \times 250\ \mathrm{h}} \times 5\ \mathrm{ms} \times 27.3\ \mathrm{ms} \times 12.9\ \mathrm{s} = 2 \times 10^{-18}.$$

(The factor 50 accounts for the resolution of the position sensitive detector used in the experiment.) This exceedingly small number holds for a background distribution which is described by Poisson's distribution.

The assignment of the decay chain to a special group of nuclei can be investigated by use of the ideas as given in Chap. 4 and 5. In [16] a slightly different method was applied by using the values of the likelihood function.

9. Summary

The mathematical relations needed for a critical analysis of results obtained by a correlation technique are presented in detail. Two typical applications are given: The nucleus ^{214}Th could be identified and the decay properties of this nucleus could be confirmed by a correlation analysis. The probability for an event chain as observed in the recent discovery of element 109 to be produced by random background is evaluated.

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